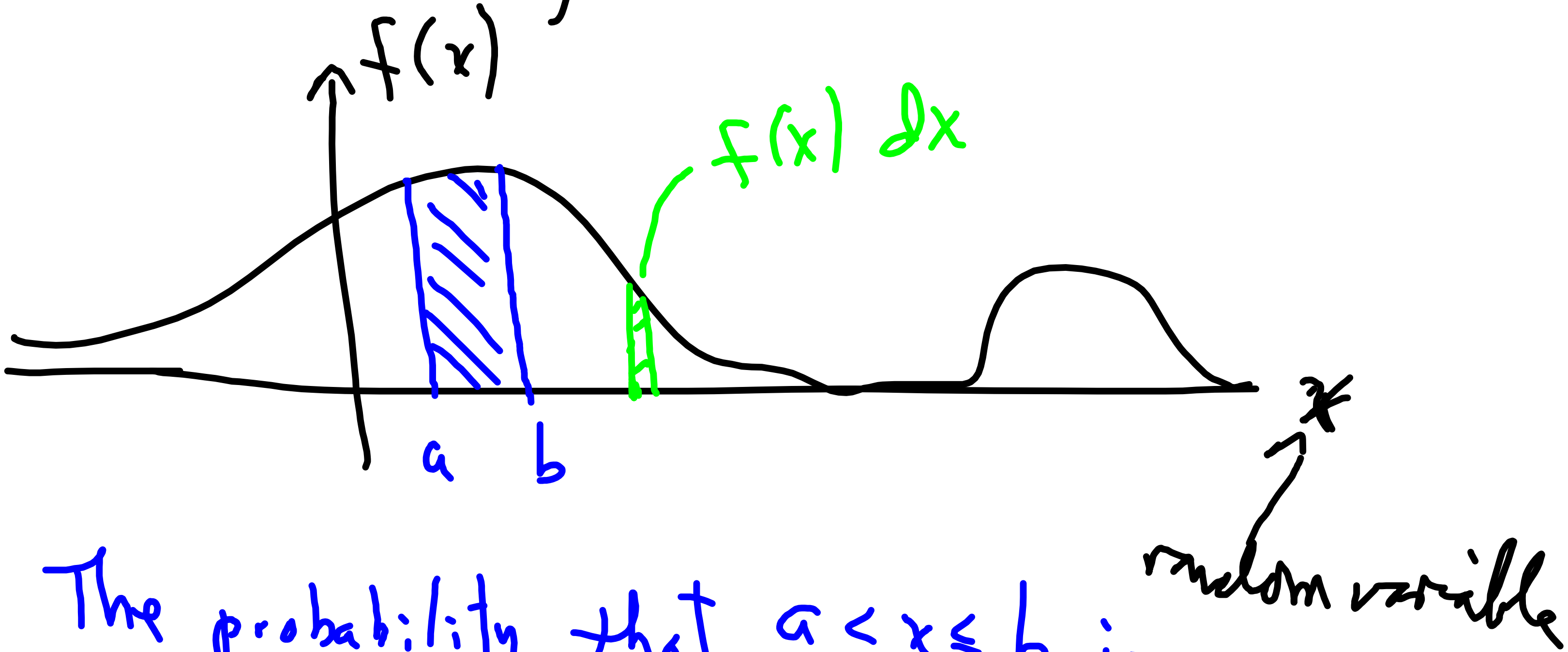
# Review of basic statistics.

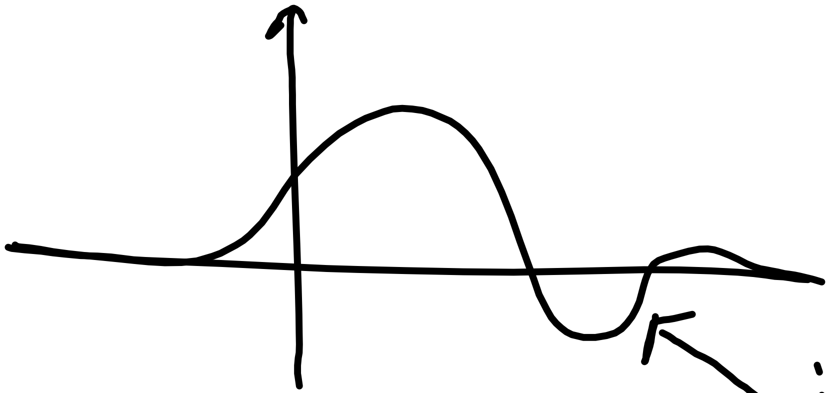
## Probability density function.




The probability that $a \le x \le b$ is

$$\int_a^b f(x)\,dx.$$

What is $\int_{-\infty}^{\infty} f(x)\,dx$ ? One.

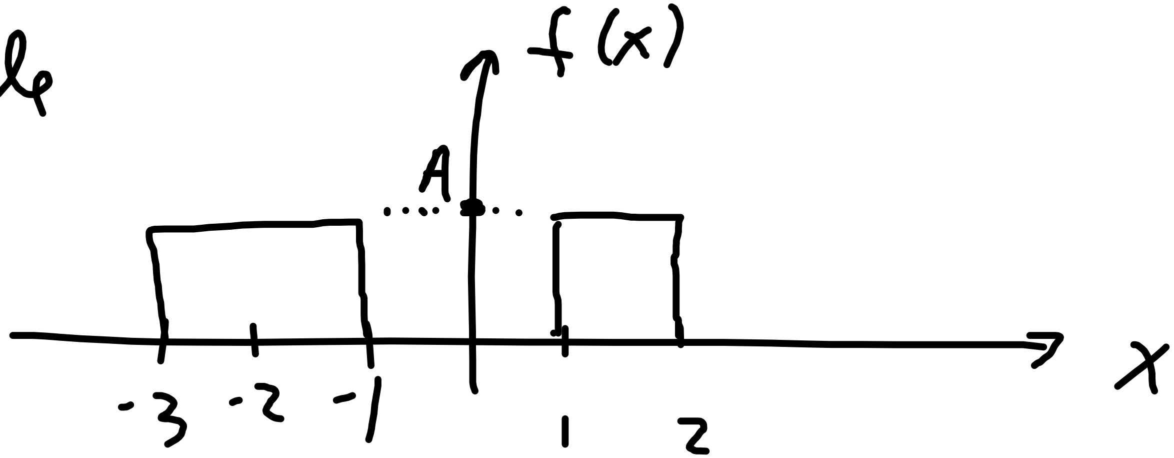


Expected values.

$$E(x) = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$= \text{"}\bar{x}\text{"}, \text{"}\mu_x\text{"}, \text{"mean"}$$

# Example

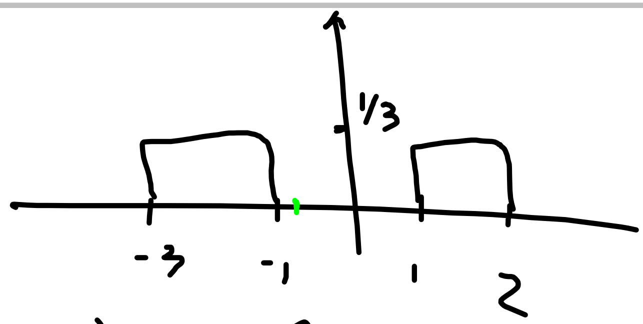



What is A? (total area = $(2+1)A = 3A = 1$)

$$\int_{-\infty}^{\infty} f(x)\,dx = \text{left} + \text{right}$$

$$= 2A + 1A$$

So $A = 1/3$




$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-3}^{-1} x \tfrac{1}{3}\,dx + \int_{1}^{2} x \tfrac{1}{3}\,dx$$

$$= \frac{1}{3}\frac{x^2}{2}\Big|_{-3}^{-1} + \frac{1}{3}\frac{x^2}{2}\Big|_{1}^{2}$$

$$= \frac{1}{3}\cdot\frac{1}{2} - \frac{1}{3}\frac{9}{2} + \frac{1}{3}\frac{4}{2} - \frac{1}{3}\frac{1}{2}$$

$$= \frac{1}{6}\left[1 - 9 + 4 - 1\right] = -5/6$$

$$\mathcal{E}(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\mathcal{E}(\sin x) = \int_{-\infty}^{\infty} \sin x \; f(x)\, dx$$

$$\mathcal{E}(F(x)) = \int_{-\infty}^{\infty} F(x)\, f(x)\, dx$$

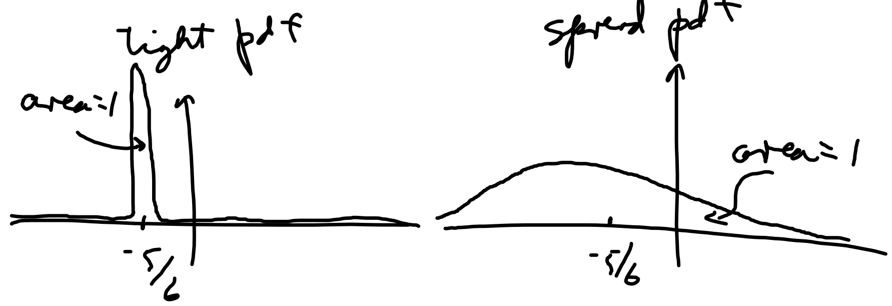
$$\mathcal{E}(x^2) = \int_{-3}^{-1} x^2 \tfrac{1}{3}\, dx + \int_{1}^{2} x^2 \tfrac{1}{3}\, dx = \overline{x^2}$$

$$= \frac{1}{3} \left. \frac{x^3}{3} \right|_{-3}^{-1} + \frac{1}{3} \left. \frac{x^3}{3} \right|_{1}^{2}$$

$$= \frac{1}{9}\left[-1 - (-27)\right] + \frac{1}{9}\left[8 - 1\right]$$

$$= \frac{1}{9}\left[-1 + 27 + 8 - 1\right] = \frac{33}{9} = 11/3 = \overline{x^2}$$

## Standard deviation:




Objective: to estimate the variation of $x$ from its mean.

$\mathcal{E}[x-\bar{x}]$ should be zero, because $x$ to the left balances $x$ to the right.

$$\mathcal{E}[x-\bar{x}] = \int_{-\infty}^{\infty} (x-\bar{x})\, f(x)\, dx$$

$$= \underbrace{\int_{-\infty}^{\infty} x\, f(x)\, dx}_{\bar{x}} - \int_{-\infty}^{\infty} \bar{x}\, f(x)\, dx$$

$$\bar{x} - \bar{x} \underbrace{\int_{-\infty}^{\infty} f(x)\, dx}_{1}$$

$$= 0$$

Look at $\mathcal{E}[(x-\bar{x})^2]$, to keep negatives from cancelling positives.

$$\mathcal{E}[(x-\bar{x})^2] = \int_{-\infty}^{\infty} (x-\bar{x})^2 \, f(x)\, dx$$

$$= \int_{-3}^{-1} \left(x - \left[-\tfrac{5}{6}\right]\right)^2 \frac{1}{3}\, dx + \int_{1}^{2} \left(x - \left[-\tfrac{5}{6}\right]\right)^2 \frac{1}{3}\, dx$$

This is called the "variance"

The standard deviation is the square root.

$$\text{StDev} = \sqrt{\int_{-\infty}^{\infty} (x-\bar{x})^2 f(x)\, dx} = \sigma$$

$$\text{Variance} = \sigma^2$$

Easier formula for variance.

$$\sigma^2 = E\left[(x-\bar{x})^2\right] = \overline{x^2} - \bar{x}^2$$

$$\overline{x^2} = 11/3 \qquad \bar{x}^2 = \left(-\frac{5}{6}\right)^2$$

$$= \frac{11}{3} - \frac{25}{36} = \frac{132 - 25}{36}$$

$$\sigma^2 = \frac{107}{36} \qquad \sigma = \frac{\sqrt{107}}{6}$$

Easier formula for variance.

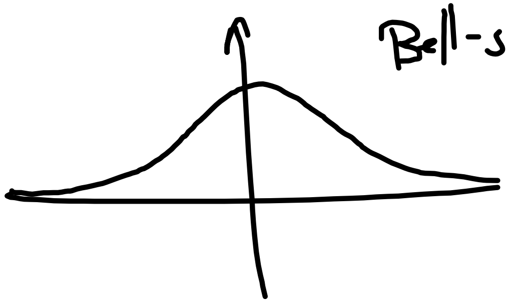
$$\sigma^2 = \mathcal{E}\left[(x-\bar{x})^2\right] = \overline{x^2} - \bar{x}^2$$

Why:
$$\mathcal{E}\left[(x-\bar{x})^2\right] = \mathcal{E}\left[x^2 - 2x\bar{x} + \bar{x}^2\right]$$

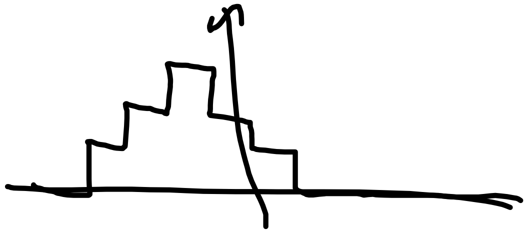
$$= \mathcal{E}\left[x^2\right] - \mathcal{E}\left[2x\bar{x}\right] + \mathcal{E}\left[\bar{x}^2\right]$$

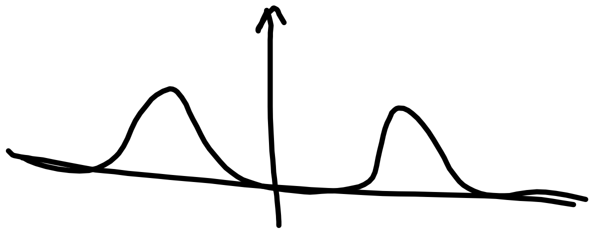
$$\overset{\shortparallel}{\overline{x^2}} \qquad -2\bar{x}\,\underbrace{\mathcal{E}(x)}_{\bar{x}} \qquad \overset{\shortparallel}{\bar{x}^2}$$

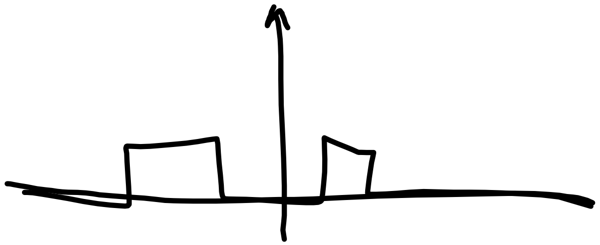
( Question: ~~wh~~ examples of pdf )



Bell-shaped

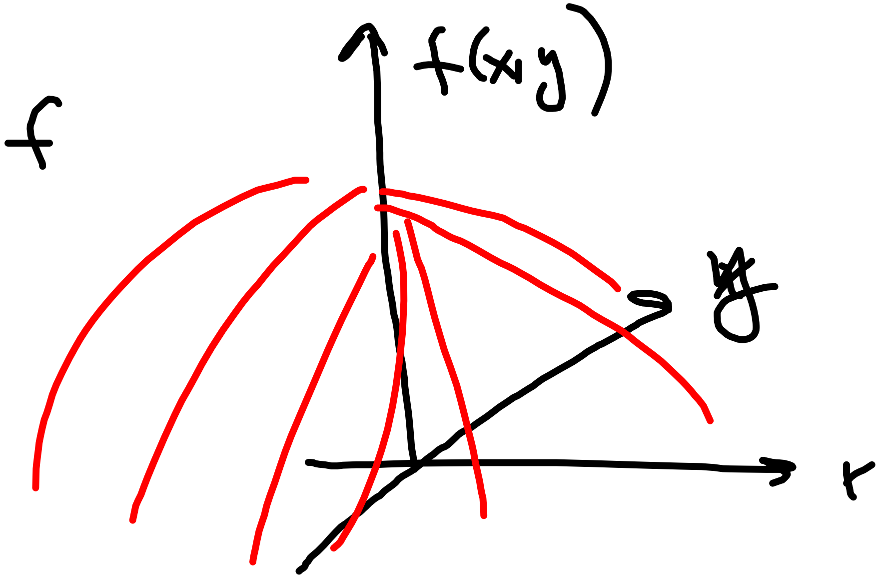






Two variables $x, y$

pdf




tent, umbrella over xy plane.

prob that $[a \le x \le b,\ c \le y \le d] = \int_{x=a}^{b} \int_{y=c}^{d} f(x,y)\,dx\,dy$

$$\mathcal{E}\{x\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f(x,y)\,dx\,dy$$

$$\mathcal{E}\{y\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\, f(x,y)\,dx\,dy$$

$$\mathcal{E}\{xy\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\,dx\,dy.$$

Independence.

pdf for $x$ : $f_X(x)$

pdf for $y$ : $f_Y(y)$

If $x$ & $y$ are independent,

$$f(x,y) = f_X(x)\, f_Y(y)$$

Given $x$ & $y$ are independent :

$$\mathcal{E}\{xy\} = \mathcal{E}[x]\,\mathcal{E}[y]$$

$$\int_{x=-\infty}^{\infty}\int_{y=-\infty}^{\infty} xy\, f(x,y)\,dy\,dx$$

$$= \iint xy\, f_X(x)\, f_Y(y)\,dy\,dx$$

$$= \int_{x=-\infty}^{\infty} x\, f_X(x)\,dx \int_{-\infty}^{\infty} y\, f_Y(y)\,dy$$

$$= \mathcal{E}(x) \qquad \mathcal{E}(y)$$