Central Limit Theorem

$X_1, X_2, X_3, \ldots, X_n, \ldots$

ind., ident. density (i.i.d.)

Then

~~is~~ (informally) $\sum_{i=1}^{n} X_n$ is Gaussian in the limit as $n \to \infty$

$$\sum_{i=1}^{n} X_i$$

mean? $E\left[\sum X_i\right] = \sum_{i=1}^{n} E(X_i) = n\mu_x$

variance?

var(sum of X_i) = sum {var(X_i)} + 2 sum{ cov(X_i, X_j)}

from previous lecture, p.2 extended to n variables. But these are independen for i not equal to j, so the covariances are zero. So the variance of the sum is the sum of the variances, or

(n) times (the variance of a single X_i).

Because they're independent, cov = 0, $= \sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \cdot = n\sigma_x^2$

So I have told you

$$\sum_{i=1}^{n} X_i \to \text{Gaussian}$$

$$\text{mean} \to n\mu_x$$

$$(\text{st dev})^2 \to n\sigma_x^2 \qquad (\text{st dev}) \to \sqrt{n}\,\sigma_x$$

GARBAGE! $n\mu_x \to \infty$

$\sqrt{n}\,\sigma_x \to \infty$

Restate more carefully

$$\frac{\sum_{i=1}^{n} X_i}{n} = \text{simple average}$$

mean $\longrightarrow \mu_X$

st dev $= \dfrac{\sigma_X}{\sqrt{n}} \to 0$

note $E(aX) = a\,E(X)$

$\sigma_{aX} = |a|\,\sigma_X$

(Theorem 2.12, 2.14 in the book.)

Homework #1.

X has mean 5.

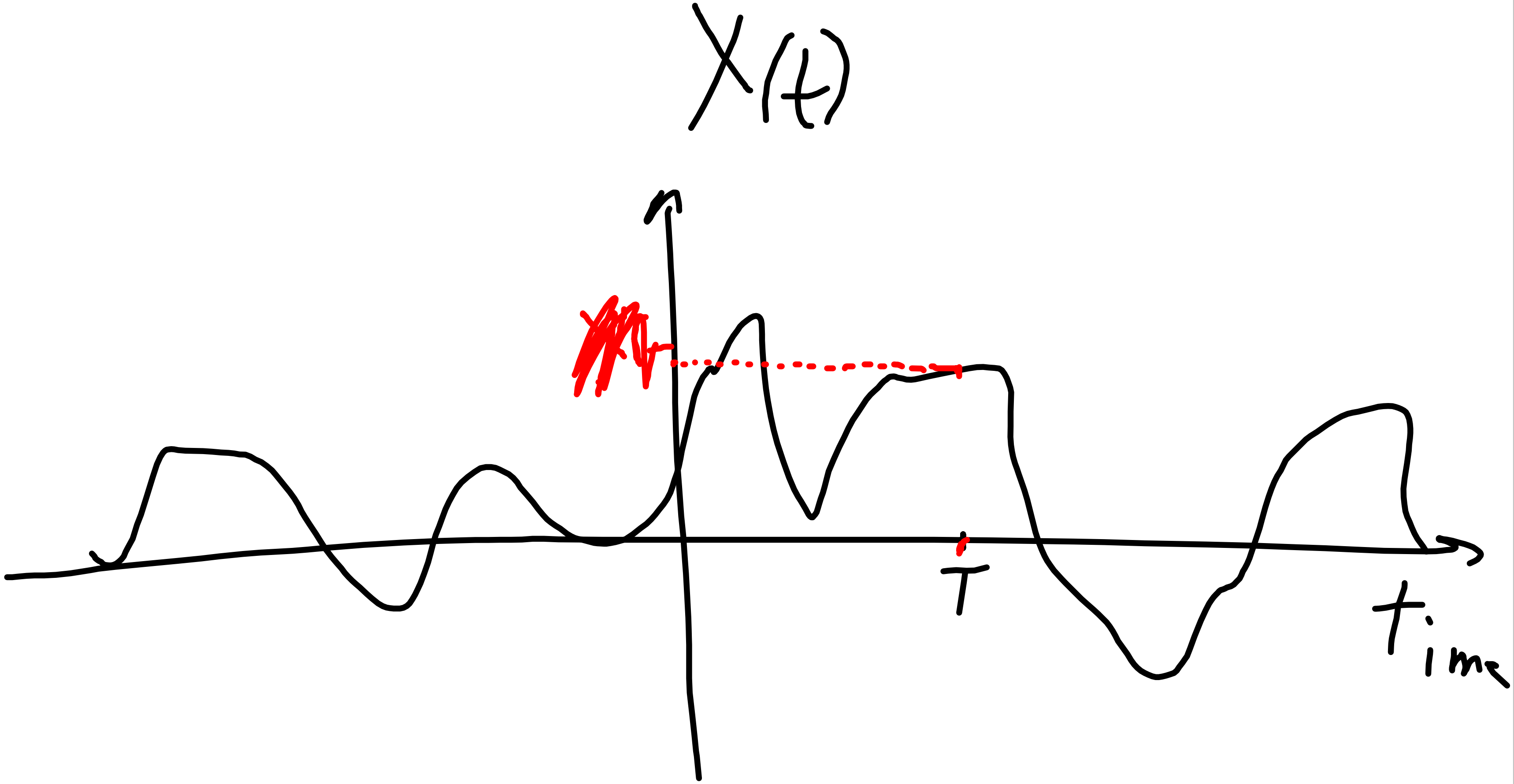
X has st. dev. 7.

What is mean $\mathcal{E}\left[\frac{X-5}{7}\right]$? 0?

What is $\sigma^2 = \mathcal{E}\left[\left(\frac{X-5}{7} - (\text{ans above})\right)^2\right]$?

1?

RANDOM PROCESSES


X(t)
T
time