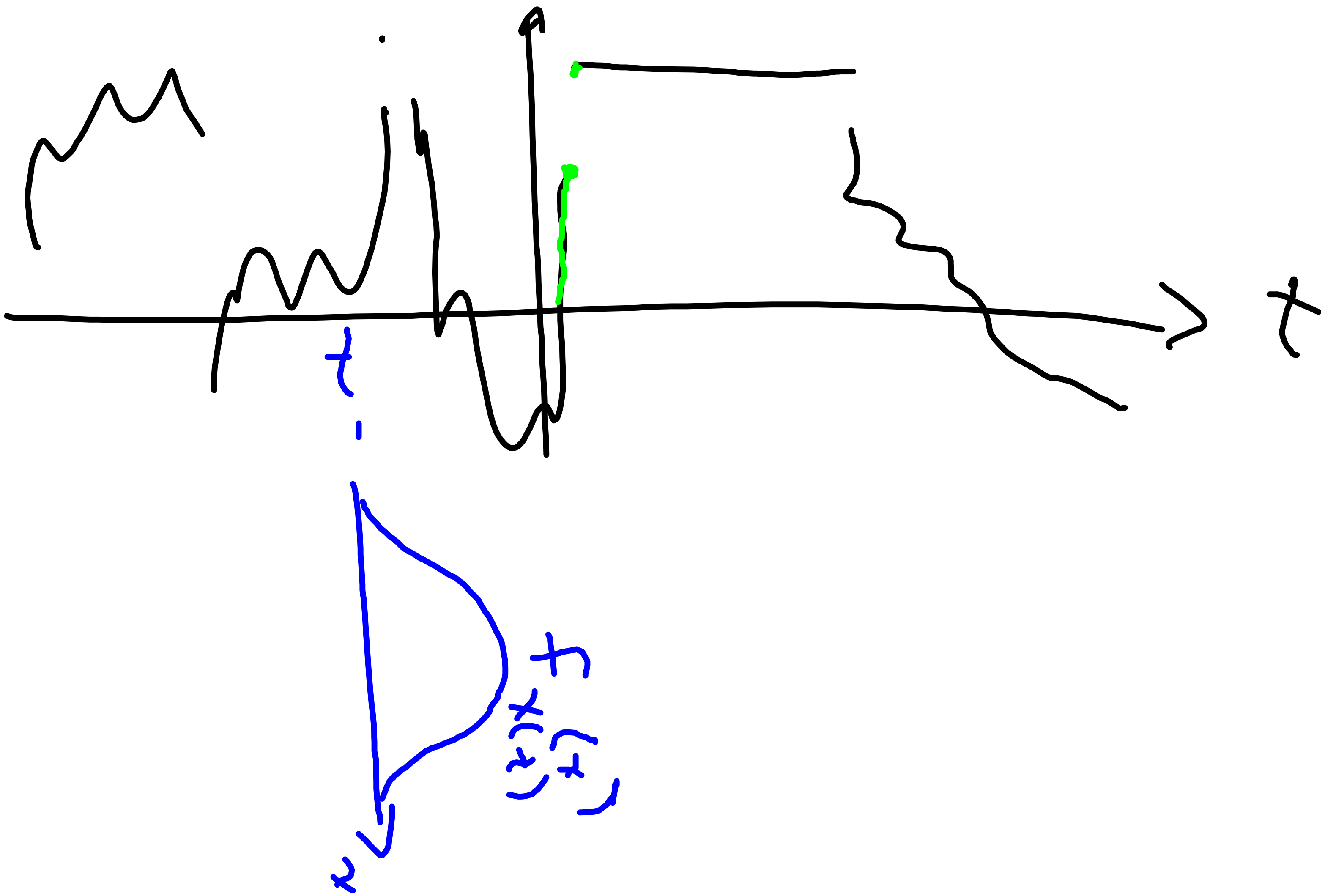
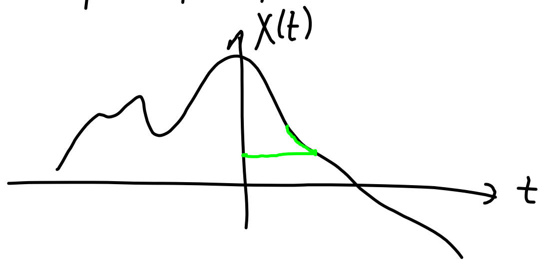
Random Process $X(t)$



Continuity of $X(t)$



continuity $f(t) \to f(t_0)$ as $t \to t_0$ $\qquad$ $f(t) - f(t_0) \to 0$ as $t \to t_0$

"$X(t)$ is continuous if $X(t) \to X(t_0)$ as $t \to t_0$, with probability one,"

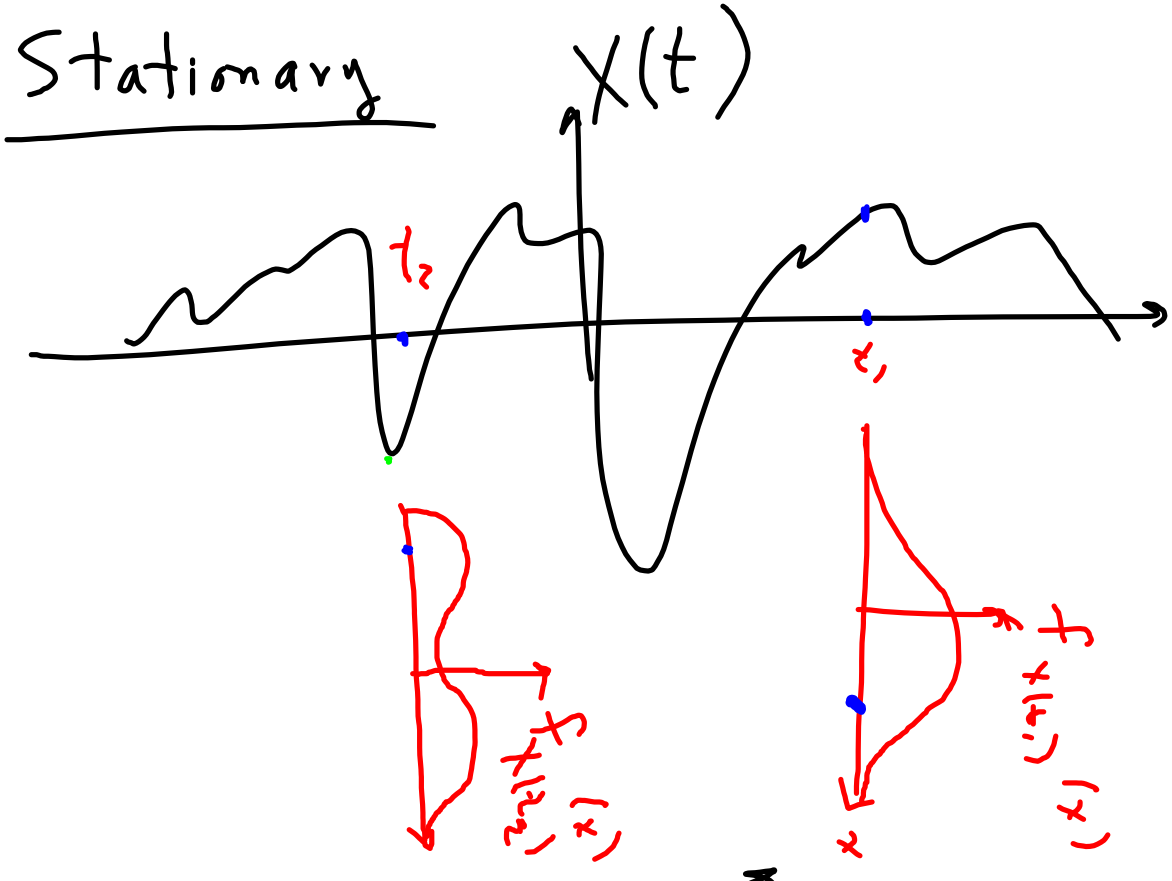
$$E\left\{\left[\lim_{t \to t_0} X(t) - X(t_0)\right]^2\right\} = 0$$

"$X(t)$ is mean-square continuous," (at t_0)

"$X(t)$ is mean-square differentiable (at t_0)"

$$E\left\{\left(\frac{X(t_0+\Delta t) - X(t)}{\Delta t} - X'(t_0)\right)^2\right\} = 0$$

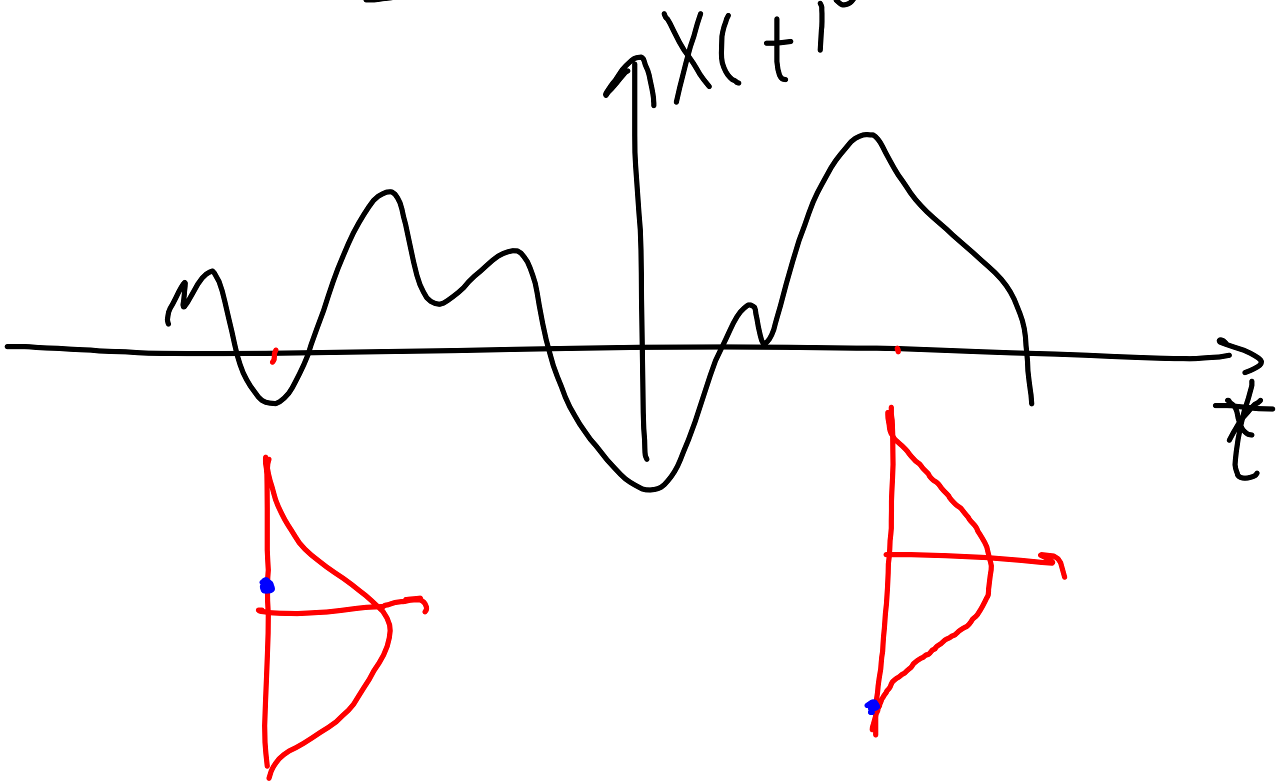
Stationary



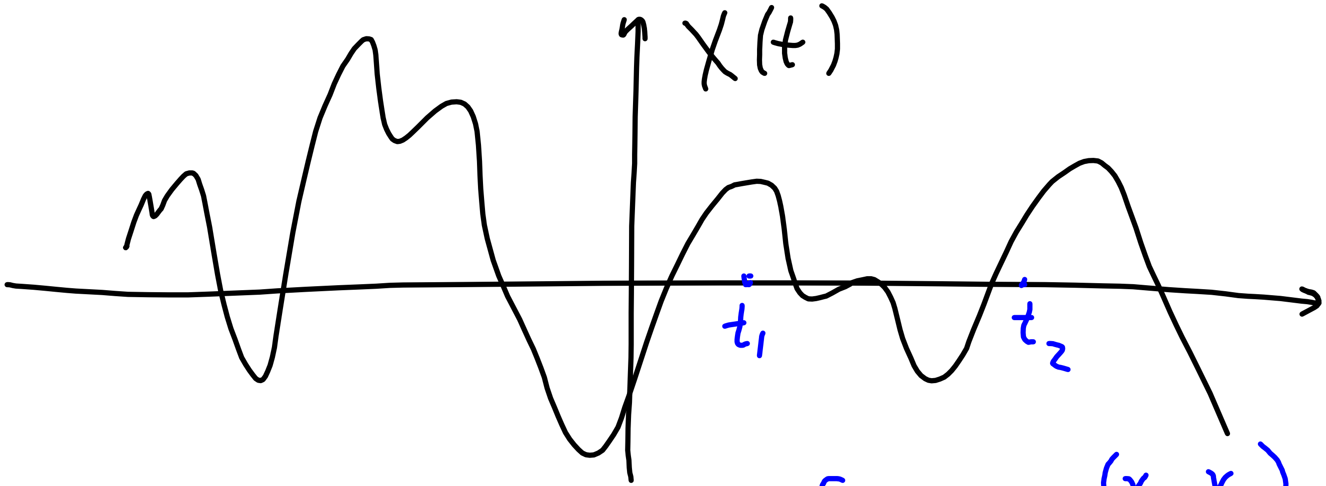
These are different,

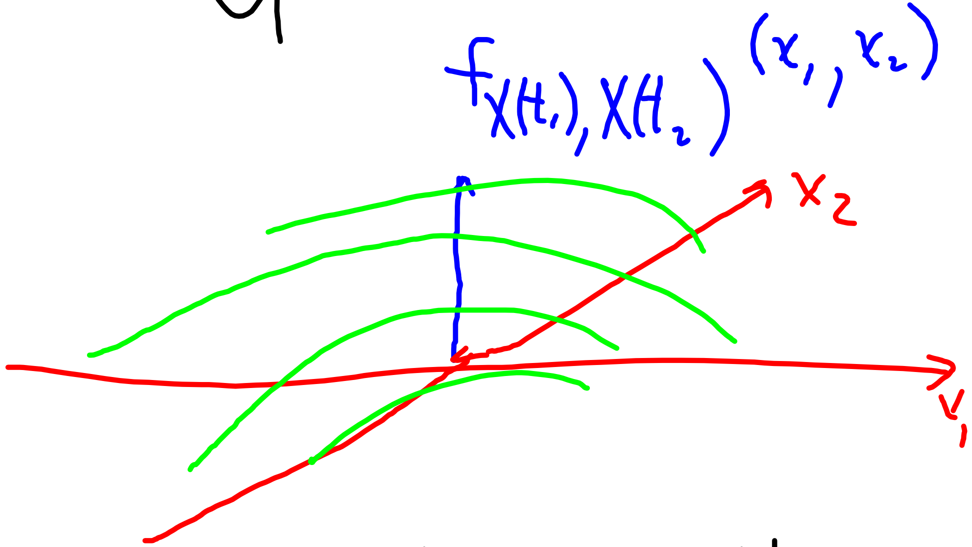
So $X(t)$ is not stationary.

This ~~is~~ one <u>is</u> stationary.



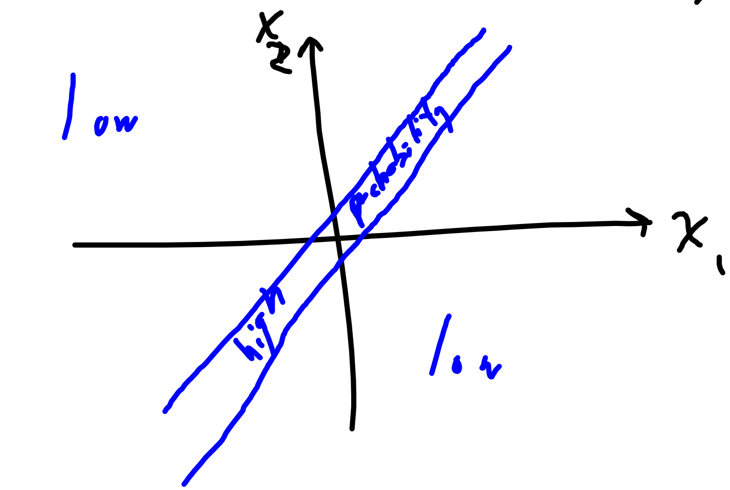
$f_{X(t)}(x)$ does not depend on t.





It would look like this if $t_1 >> t_2$.

If $X(t)$ is continuous & $t_1 \approx t_2$,
then $X(t_1) \approx X(t_2)$ almost certainly.



So, it is not reasonable to demand $f_{X(t_1),X(t_2)}(x_1,x_2)$ be independent of t_1 & of t_2. Instead,

$f_{X(t_1),X(t_2)}(x_1,x_2)$ depends only on $(t_1 - t_2)$,
(for stationary)

$$\mathcal{E}\left[X(t) \right] = \mu\cancel{(t)}$$

$$\mathcal{E}\left[X(t)^2 \right] = \overline{X^2\cancel{(t)}} \quad \sigma\cancel{(t)}$$

$$\mathcal{E}\left[X(t_1)\, X(t_2) \right] \text{ depends on } (t_1 - t_2)$$

Fact 1. Independence implie
autocovariance = 0.

(Remember autocov = autocorr. if
process is ~~z~~ero-mean.)

Fact 2. Zero autocov AND Gaussian
implies independence

Stationary process

$$E\left[\left(X(t_1)-\mu\right)\left(X(t_2)-\mu\right)\right] = R_X(\tau)$$

depends only on $t_1 - t_2 =$ "τ".

"autocorrelation" or "autocovariance".

Actually, this only depends on $|t_1 - t_2|$.

Homework

$$R_X(\tau) = \frac{\cos \pi \tau}{\tau^2 + 1}$$ must be an even function.

$$\frac{\cos \pi \tau}{\tau^3 + 1}$$ impossible

$\sin \pi \tau$ "

$e^{-\tau}$ "

$$R_X(\tau) = E[X(t)\,X(t+\tau)]$$

$$= E[(X(t)-\mu)(X(t+\tau)-\mu)]$$

$$R_X(0) = \sigma^2$$

$$\rho(\tau) = \frac{R_X(\tau)}{R_X(0)}$$