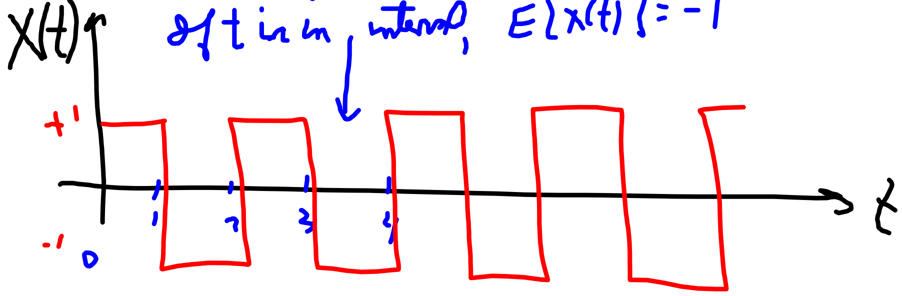
Random Process $\text{mean}(t) = E[X(t)]$

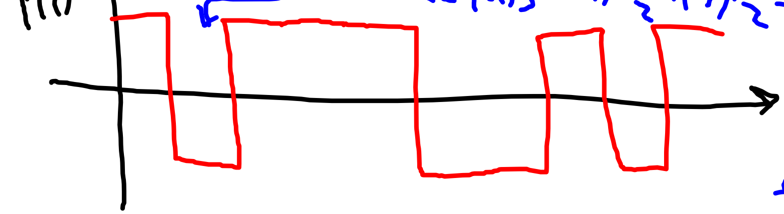
#1 Deterministic

If t in interval, $E[X(t)] = -1$



If t in interval, $E[X(t)] = 1$

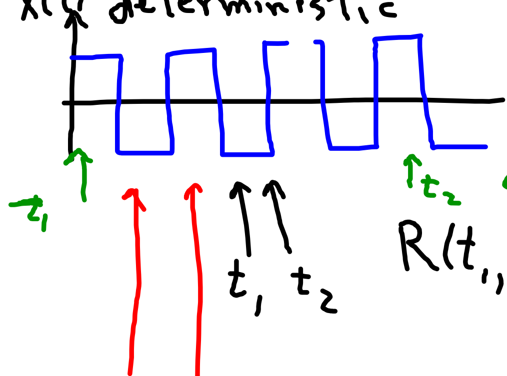
If t is in interval $E[Y(t)] = (1)\cdot\frac{1}{2} + (-1)\cdot\frac{1}{2} = 0$



$= 0$ everywhere

Autocorrelation: $R(t_1,t_2) = E[X(t_1)X(t_2)]$

$X(t)$ deterministic



$E[X(t_1)X(t_2)] = X(t_1)X(t_2)$

$E[X(t_1)X(t_2)] = (+1)(+1) = 1$

$R(t_1,t_2) = (-1)(-1) = 1$

$t_1 \quad t_2 \quad R(t_1,t_2) = (-1)(+1) = -1$

$$\text{Autocovariance} = E\left[\left(X(t_1) - E[X(t_1)]\right)\left(X(t_2) - E[X(t_2)]\right)\right]$$

$$= C(t_1, t_2)$$

$X(t)$ "white noise"

zero mean : $E[X(t)] = \text{mean}(t) = 0$

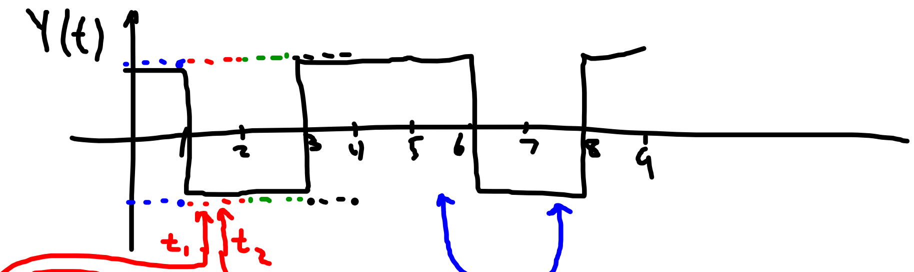
$$R(t_1, t_2) = E[X(t_1)X(t_2)] = E[X(t_1)]E[X(t_2)]$$

$E[X(t_1)] = 0$, $E[X(t_2)] = 0$

white noise $\Longleftrightarrow$ independent

if $t_1 \neq t_2$, $= 0$

$$R(t_1, t_1) = E[X(t_1)X(t_1)] = \text{var}(X(t_1)) = \sigma^2$$



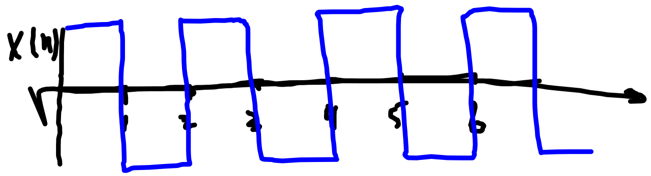
$$E[X(t_1)X(t_2)] = R(t_1,t_2) = (+1)(+1)\cdot\tfrac{1}{4} + (+1)(-1)\cdot\tfrac{1}{4} + (-1)(+1)\cdot\tfrac{1}{4} + (-1)(-1)\cdot\tfrac{1}{4}$$

$$= 0$$

$$E[X(t_1)X(t_2)] = R(t_1,t_2) = (-1)(-1)\cdot\tfrac{1}{2} + (+1)(+1)\cdot\tfrac{1}{2} = 1$$

If t_1 & t_2 belong to the same interval, $R(t_1,t_2) = 1$

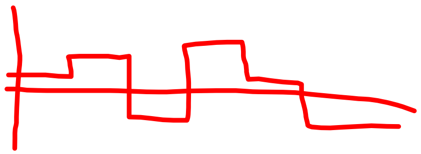
——— different ", $R(t_1,t_2) = 0$



$$X(n) = \begin{cases} +1 & \text{if } n=0 \\ -1 & \text{if } n=1 \\ +1 & \text{if } n=2 \\ -1 & \text{if } n=3 \end{cases} = (-1)^n$$

(deterministic)

W(n) white noise

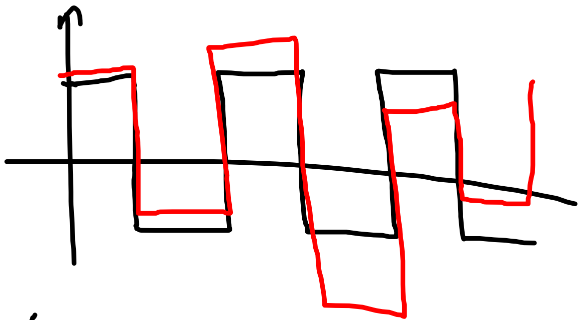


$X(n) + W(n) = \hat{X}(n)$

mean

$$E[\hat{X}(n)] = E[X(n) + W(n)]$$
$$= E[X(n)] + E[W(n)]$$
$$= (-1)^n + 0$$

$\tilde{X}(n) = X(n) + W(n)$



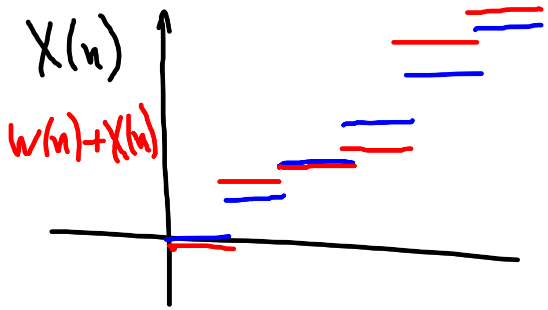
$R(n_1, n_2) = E[\tilde{X}(n_1)\tilde{X}(n_2)]$

$= E[X(n_1)X(n_2) + X(n_1)W(n_2) + X(n_2)W(n_1) + W(n_1)W(n_2)]$

$X(n_1)X(n_2)$ $\quad X(n_1)E[W(n_2)]$ $\quad 0$ $\quad 0$ if $n_1 \neq n_2$, $E[W(n)^2]$ if $n_1 = n_2$ $= \sigma_W^2$

$X(n_1)E[W(n_2)] = 0$

$R(n_1, n_2) = (-1)^{n_1+n_2} + \sigma_W^2 \delta_{n_1, n_2}$

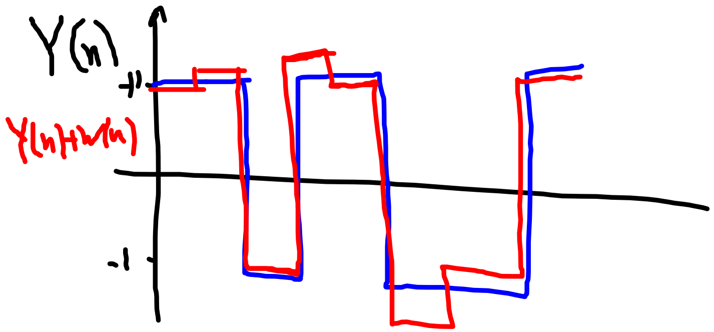


$$X(n) = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ 2 & \text{if } n=2 \\ \vdots & \end{cases} = n$$

mean $E\{X(n)+W(n)\} = n + 0 = n$

$$R(n_1, n_2) = E[\{X(n_1)+W(n_1)\}\{X(n_2)+W(n_2)\}]$$

$$= n_1 n_2 + \sigma_w^2 \delta_{n_1,n_2}$$



mean $= E[\,Y(n)+W(n)\,] = \underbrace{E[\,Y(n)\,]}_{=0} + \underbrace{E[\,W(n)\,]}_{=0} = 0$

autocorrelation

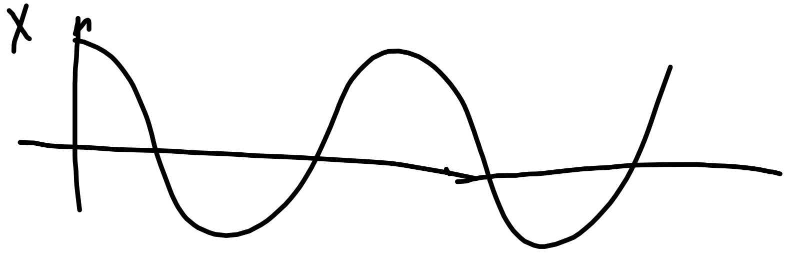
$$R(n_1,n_2) = E\big[\{Y(n_1)+W(n_1)\}\{Y(n_2)+W(n_2)\}\big]$$

$$= \underbrace{E[\,Y(n_1)Y(n_2)\,]}_{R_Y(n_1,n_2)} + \underbrace{E[\,Y(n_1)\,W(n_2)\,]}_{\text{assume independence}} + E[\,Y(n_2)\cdot W(n_1)\,]$$

$$+ \underbrace{E[\,W(n_1)\,W(n_2)\,]}_{\sigma_W^2\,\delta_{n_1,n_2}}$$

$$E[\,Y(n_1)\,]\,E[\,W(n_2)\,] \qquad E[\,Y(n_1)\,] = 0, \quad E[\,W(n_2)\,] = 0$$

$X(t) = \cos \omega t$ deterministic



$\text{mean}(t) = E\{X(t)\} = E[\cos \omega t] = \cos \omega t$

$R(t_1, t_2) = E\{\cos \omega t_1 \cos \omega t_2\} = \cos \omega t_1 \cos \omega t_2$

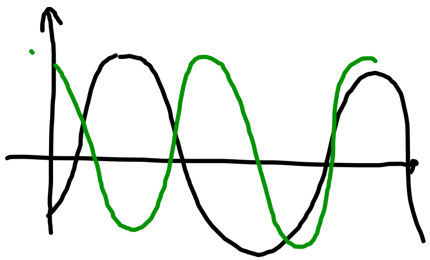
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

$= \dfrac{\cos \omega(t_1+t_2)}{2} + \dfrac{\cos \omega(t_1-t_2)}{2}$

$X(t) = \cos(\omega t + \Theta)$



$$\text{mean}(t) = E\{X(t)\} = E\{\cos(\omega t + \theta)\}$$

$$= \int_{\theta=0}^{2\pi} f_\Theta(\theta) \cos(\omega t + \theta)\, d\theta = 0$$

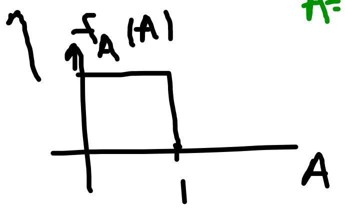
$$R(t_1, t_2) = E[\cos(\omega t_1 + \theta)\cos(\omega t_2 + \theta)]$$

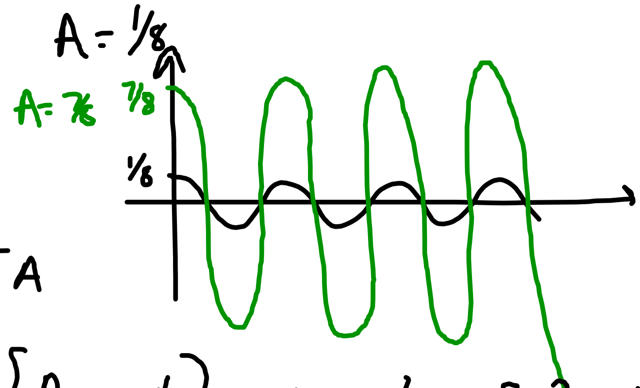
$$= \int_0^{2\pi} f_\Theta(\theta) \underbrace{\cos(\omega t_1 + \theta)\cos(\omega t_2 + \theta)}_{\frac{\cos[\omega(t_1+t_2)+2\theta]}{2} + \frac{\cos[\omega(t_1-t_2)]}{2}}\, d\theta$$

$$\frac{\cos[\omega(t_1+t_2)+2\theta]}{2} \rightarrow 0$$

$$\frac{\cos[\omega(t_1-t_2)]}{2} \rightarrow \frac{\cos \omega(t_1-t_2)}{2}(1)$$

$X(t) = A \cos \omega t$





$$\text{mean}(t) = E[A \cos \omega t] = \cos \omega t \, E[A] = \frac{1}{2} \cos \omega t$$

$$R(t_1, t_2) = E[A \cos \omega t_1 \, A \cos \omega t_2] = \cos \omega t_1 \cos \omega t_2 \underbrace{E[A^2]}_{1/3}$$