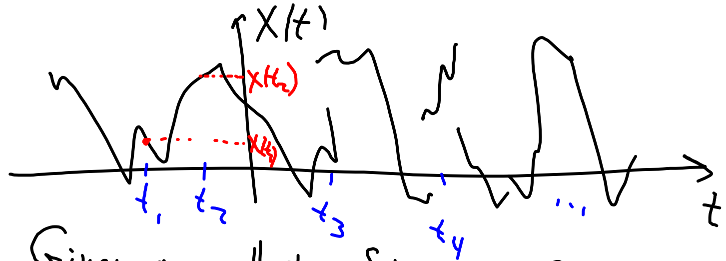
Random Process



Given any collection $\{t_1, t_2, \dots, t_n\}$, the values $X(t_1), X(t_2), \dots, X(t_n)$ are random variables. They have a joint distribution fcn.

for X, Y, joint $f_{XY}(x, y)$

$$f_{X(t_1), X(t_2), X(t_3), \dots X(t_n)}(x_1, x_2, x_3, \dots, x_n)$$

Why not $f_{\dots}(x(t_1), x(t_2) \dots, x(t_n))$

?

Because $\text{prob}[-1 \le X(t_1) \le 1]$

$$= \int_{x_n=-\infty}^{\infty} \cdots \int_{x_2=-\infty}^{\infty} \int_{x_1=-1}^{+1} f_{\dots}(x_1, x_2, \dots, x_n)\, dx_1 \dots dx_2 \cdots dx_n$$

Properties, Questions.

$$f_{X(t_1),X(t_2),X(t_3)}(x_1, x_2, x_3) \quad \text{is a marginal}$$

$$\int_{x_4=-\infty}^{\infty} f_{X(t_1),X(t_2),X(t_3),X(t_4)}(x_1, x_2, x_3, x_4)\, dx_4$$

at time t_1, $f_{X(t_1)}(x_1)$

$$\int_{-\infty}^{\infty} f_{X(t_1)}(x_1)\, dx_1 = 1$$

$$\int_{-\infty}^{\infty} f_{X(t_1)}(x_1) \cdot x_1 \, dx_1 = E[X(t_1)]$$

This is a number that depends on t_1.

$$= \mu_X(t_1)$$

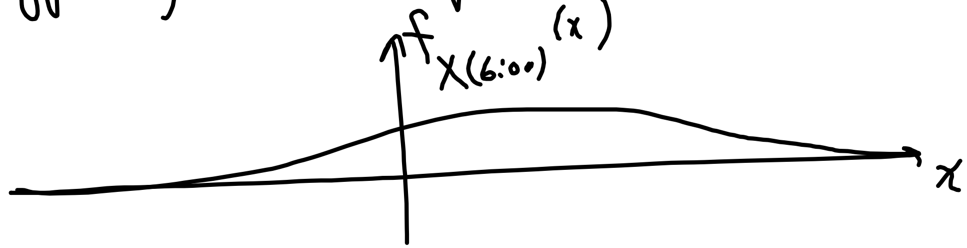
$$\int_{-\infty}^{\infty} f_{X(t_1)}(x_1) \left[x_1 - \mu_X(t_1)\right]^2 dx_1 = \sigma_X(t_1)^2$$

Question: does pdf have to be defined for all times t, ?

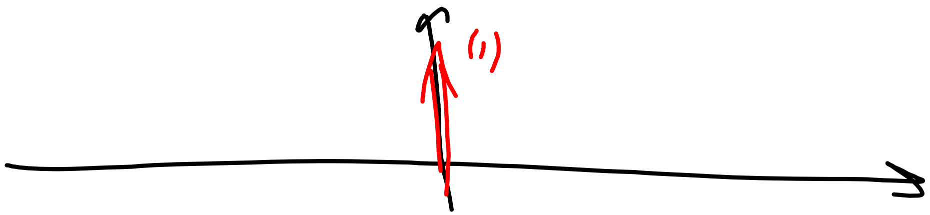
Answer   example of pdf

$$f_{X(t_1),\,X(t_2)}(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - \frac{1}{2}\left(\frac{x_2-\mu_2}{\sigma_2}\right)^2}$$

Suppose you have a pdf for X(6:00 pm tonight)



Suppose power goes off at 5:00 pm.; so ~~Pdf~~ pdf for 6:00 pm changes to



This is a conditional pdf for 6:00 pm, given that power went off at 5:00 pm.

$$f_{X(6:00)|X(5:00)=60} = \frac{f_{X(6)\, X(5)}}{f_{X(5:00)}}$$

2 time joint pdf

$$f_{X(t_1),X(t_2)}(x_1,x_2)$$

$$\mathcal{E}\{X(t_1)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 f_{X(t_1),X(t_2)}(x_1,x_2)\,dx_1\,dx_2$$

$$\mathcal{E}\{X(t_1)X(t_2)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1),X(t_2)}(x_1,x_2)\,dx_1\,dx_2$$

This is the "correlation" between $X(t_1)$ and $X(t_2)$,

$$= \text{corr}[X(t_1), X(t_2)] = \text{corr}(t_1, t_2)$$

This is "autocorrelation".

How do we come up with these pdf's?

① Have a model. (Later)

② Average over past data.

<u>Assumptions.</u>

1. Stationarity: the pdf's don't change over time.

2. Ergodicity:

(You'd expect to be able to estimate statistical parameters by repeating the experiment over and over, & taking averages), but if you only get 1 chance, then postulate that you can use time-history averages ~~of one~~ to estimate population means.