Have $X(t)$ F.T. of $X_T(t) = \hat{X}_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-i2\pi f t} dt$

auto-corr $R_x(\tau) = E[X(t)X(t+\tau)]$

PSD $S_x(f) =$ F.T. of $R_x(\tau) = \int_{-\infty}^{\infty} R_x(\tau) e^{-i2\pi f \tau} d\tau$

Elaborate on why $R_X(\tau)$ estimate is unreliable for large τ.

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{i2\pi f\tau} df$$ (inv FT.)

$R_X(\tau)$: true, theoretical, actual

$S_X(f)$: true, theoretical, actual PSD.

BUT we estimate $R_X(\tau)$ by

$$\frac{1}{T} \quad \frac{1}{T-\tau} \int_{-T/2}^{T/2} X_T(t) X_T(t+\tau) dt$$

$\frac{1}{T}$: really is

$\frac{1}{T-\tau}$: should be

How did I get the $X(t)$ in the demo?

Difference start $X(0) = 1$

$$X(n) = X(n-1) + N(n)$$

noise: white Gaussian noise: mean zero, $\sigma_N^2 = 1$

Independent of preceding X's.

actually

$$b_1 X(n) = -b_2 X(n-1) + a_1 N(n)$$

Note to readers: In the MATLAB demo, I coded the a's as the coefficents of X(n), and the b's as the coefficients of the noise terms - just the reverse of what the lecture notes say. specestdemo assumes that the difference equation say
a_1 X(n) = - a_2 X(n-1) + b_1 N(n) .
When you try the simulations, make this change.

Analysis of $b_1 X(n) = -b_2 X(n-1) + a_1 N(n)$

$b_1 = 1$, $a_1 = 1$

$$b_1 X(n) + b_2 X(n-1) + b_3 X(n-2) \cdots$$
$$= a_1 N(n)$$

$$\boxed{b_1 X(n) = -b_2 X(n-1) + a_1 N(n)}$$

What is $E[X(n)]$?

(assume $X(n)$ is stationary.)

$$b_1 E[X(n)] = -b_2 \underbrace{E[X(n)]}_{\text{because stationary}} + a_1 \cdot 0$$

$$E[X(n)] = \frac{0}{b_1 + b_2} = 0 \quad (\text{unless } b_2 = -b_1 = -1)$$

Ground rule: $b_2 \neq -b_1$.

Difference Eq Theory:

$$b_1 x(n) = -b_2 x(n-1) + f(n)$$

Remember: $y' = ay + f(x)$ $y(0) = y_0$.

Look for $y'_{hom} = a y_{hom}$ homog. eq.,

$y'_{part} = a y_{part} + f(x)$ with initial condition.

$$y(x) = y_{hom}(x) + y_{part}(x)$$

use constants to get right initial condition.

Apply this strategy to difference eq.

① homog. equation

$$b_1 \, x(n) = -b_2 \, x(n-1) \qquad x(0) = A$$

$$\Longrightarrow \quad x(n) = -\frac{b_2}{b_1} x(n-1)$$

$x(0)$	$x(1)$	$x(2)$	$x(3)$	$\cdots$
A	$-\frac{b_2}{b_1} A$	$\left(-\frac{b_2}{b_1}\right)^2 A$	$\left(-\frac{b_3}{b_1}\right)^3 A$	$\cdots$

② Particular soln of nonhomog eq.

$$b_2 x(n) = -b_1 x(n-1) + f(n)$$

$$x(n) = -\frac{b_1}{b_2} x(n-1) + \frac{f(n)}{b_2}$$

$x(0)$	$x(1)$	$x(2)$	$x(3)$	$\dots$
0 (why not?)	$\frac{f(1)}{b_2}$	$-\frac{b_1}{b_2}\frac{f(1)}{b_2} + \frac{f(2)}{b_2}$	$\dots$	

So solution of $b_1 X(n) = -b_2 X(n-1) + a_1 N(n)$

has solutions

$$X(n) = X_{hom}(n) + X_{part}(n)$$

$$X_{hom}(n) = A,\ -\frac{b_2}{b_1}A,\ \left(-\frac{b_2}{b_1}\right)^2 A,\ \left(-\frac{b_2}{b_1}\right)^3 A$$

→ This contribution is TRANSIENT (dies out) if $|b_2| < |b_1|$.

and, after it dies out, the effect of the initial condition is gone.

So to generate a ~~random~~ stationary process, use

$$b_1 X(n) = -b_2 X(n-1) + a_1 N(n),$$

choose $|b_2| < |b_1|$,

run for awhile until the transients have died. Stationary

Note to readers: In the MATLAB demo, I coded the a's as the coefficents of X(n), and the b's as the coefficients of the noise terms - just the reverse of what the lecture notes say. specestdemo assumes that the difference equation say
a_1 X(n) = - a_2 X(n-1) + b_1 N(n) .
When you try the simulations, make this change.

So assume $b_1 X(n) = -b_2 X(n-1) + a_1 N(n)$

stationary.

What is $R_X(m)$?

$$= E[X(n+m)X(n)]$$

$$= E[X(m)X(0)]$$

$$b_1 X(n) = -b_2 X(n-1) + a_1 N(n)$$

multiply by $X(0)$, take $E\{-\}$.

$$b_1 R_X(n) = -b_2 R_X(n-1) + a_1 E\{N(n)X(0)\}$$

$E\{N(n)\}E\{X(0)\}$

if independent.

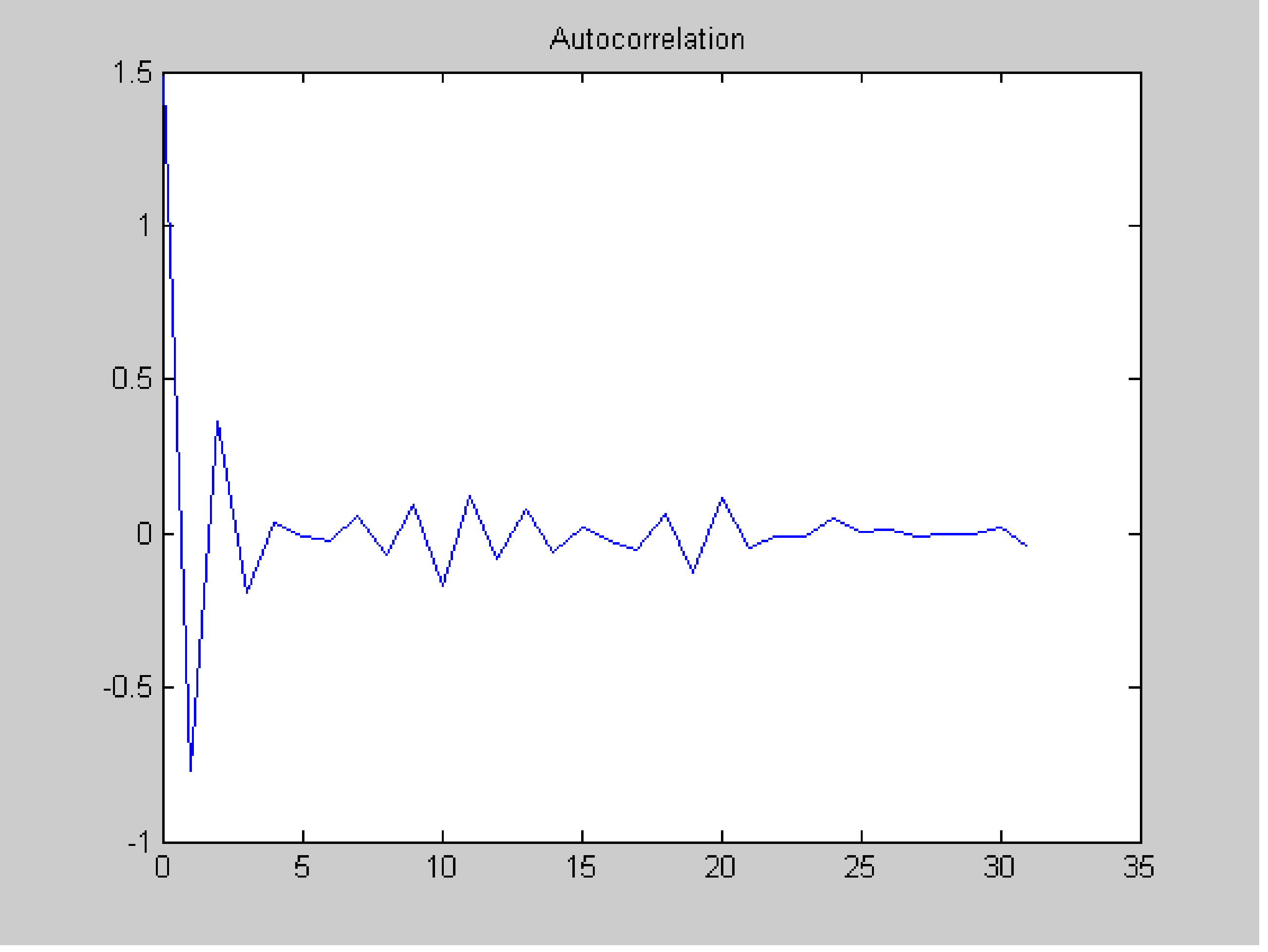
$N(n)$ is independent

of $X(0)$ if $n=1,2,3,\ldots$

$$R_X(n) = -\frac{b_2}{b_1} R_X(n-1) \quad \text{for } n>0$$

For the simulation, $b_1=1$, $b_2=.5$, $a_1=1$ $R_X(n) = -\frac{1}{2}R_X(n-1)$

Note to readers: In the MATLAB demo, I coded the a's as the coefficents of X(n), and the b's as the coefficients of the noise terms - just the reverse of what the lecture notes say. specestdemo assumes that the difference equation say
a_1 X(n) = - a_2 X(n-1) + b_1 N(n) .
When you try the simulations, take a_1 = 1, a_2 = .5, b_1 = 1, and you'll see the successive halving of the autocorrelation values.


Autocorrelation
1.5
1
0.5
0
-0.5
-1
0
5
10
15
20
25
30
35

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