Random Process.

Zero-mean, ergodic.

Auto-correlation:

$$R_x(l) = \mathbb{E}[X(t)X(t+l)]$$

$$R_x(0) = \sigma^2 \quad \text{"Power"}$$

Note $R_x(l)$ is not a random variable.

Fourier Transform: $S(f) = \int_{-\infty}^{\infty} R_x(t) e^{-i2\pi ft} dt$

Inverse Transform:

$$R_x(t) = \int_{-\infty}^{\infty} S(f) e^{i2\pi ft} df$$

In particular, $R_x(0) = \int_{-\infty}^{\infty} S(f) df$

$$\sigma^2 \quad \text{\"Power\"}$$

$S(f)$ is power spectral density
Implementation: estimation.

To estimate $R_x(t)$ the hard way,

$$\lim_{T \to \infty} \frac{1}{T - T/2} \int_{T/2}^{T} x(t) x(t+\tau) \, dt$$

$$\mathbb{E}[\text{this}] = R_x(t) \quad \text{(which is reassuring)}$$

Too much work.
Another flaw in the implementation of this formula.

\[ \frac{1}{T} \sum_{-T/2}^{T/2} X(t) X(t+\tau) d\tau \quad \text{OK.} \]

Usually interpreted as:

\[ X(t) \]

\[ X_T(t) \]

\[ \frac{1}{T} \sum_{-T/2}^{T/2} X_T(t) X_T(t+\tau) d\tau \]

You should be dividing \( \frac{1}{T-\chi_T} \) instead of \( \frac{1}{T} \).

Remark: Use \( \frac{1}{T} \), only give credibility to the \( \chi_T \) inside for \( |\tau| < T \).
To estimate $R_x(t)$ the "easy" way,
estimate F.T. ($S(f)$) first, then
inverse-transform.

Wigner
start with $x(t)$; truncate $X_T(t)$,
take F.T. of $X_T(t) \rightarrow \hat{X}_T(f)$,
magnitude-square; $|\hat{X}_T(f)|^2$;
divide by "T" (or # of time samples);

\[
\mathbb{E}\left\{ \frac{1}{T} |\hat{X}_T(f)|^2 \right\} = S(f)
\]

\[
\mathbb{E}\text{[s.w. of]} \frac{1}{T} |\hat{X}_T(f)|^2 \right\} = R_x(t)
\]

"Bartlett's Method"
class Questions

Batch

Suggested: sub sampling
Homework: Run specest demo.

When it asks for b, input [\# 7]

\[-1 < \# < 1\]  \( (ty 2) \).

When it asks for a, input [1]

Return, return, ...