ORIENTED

Random Process.

Zero-mean, ergodic.

Auto correlation:

$$R_X(|\tau|) = E\left[X(t)\, X(t+\tau) \right]$$

$$R_X(0) = \sigma^2 \quad \text{"power"}$$

Note $R_X(|\tau|)$ is not a random variable.

Fourier Transform: $S(f) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-i 2\pi f \tau}\, d\tau$

Inverse Transform

$$R_X(\tau) = \int_{-\infty}^{\infty} S(f)\, e^{i 2\pi f \tau}\, df$$

In particular, $R_X(0) = \int_{-\infty}^{\infty} S(f)\, df$

σ^2 = power

$S(f)$ is power spectral density

Implementation: estimation.

To estimate $R_X(\tau)$ the hard way,

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)\, X(t+\tau)\, dt$$

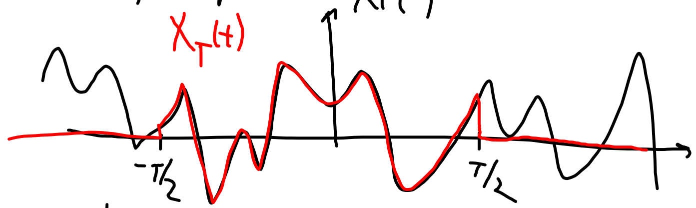
$$\mathcal{E}[\text{this}] = R_X(\tau)$$ (which is reassuring)

Too much work.

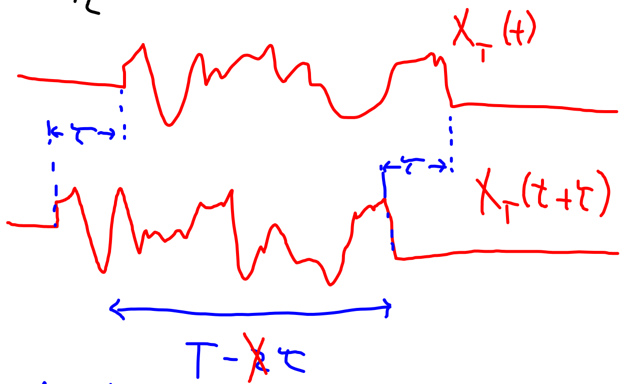
Another flaw in the implementation of this formula.

$$\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t+\tau)\,dt \qquad \text{OK.}$$

Usually interpreted as



$$\frac{1}{T}\int_{-T/2}^{T/2} x_T(t)\,x_T(t+\tau)\,dt$$



You should be dividing $\frac{1}{T-\cancel{2}\tau}$ instead of $\frac{1}{T}$.

Remedy: use $\frac{1}{T}$, only give credibility to the estimate for $|\tau| << T$.

To estimate $R_X(\tau)$ the "easy" way,

estimate F.T. ($S(f)$) first, then

inverse-transform.

<u>Wiener</u> start with $X(t)$; truncate $X_T(t)$,

take F.T. of $X_T(t) \rightarrow \tilde{X}_T(f)$,

magnitude-square; $\left|\tilde{X}_T(f)\right|^2$;

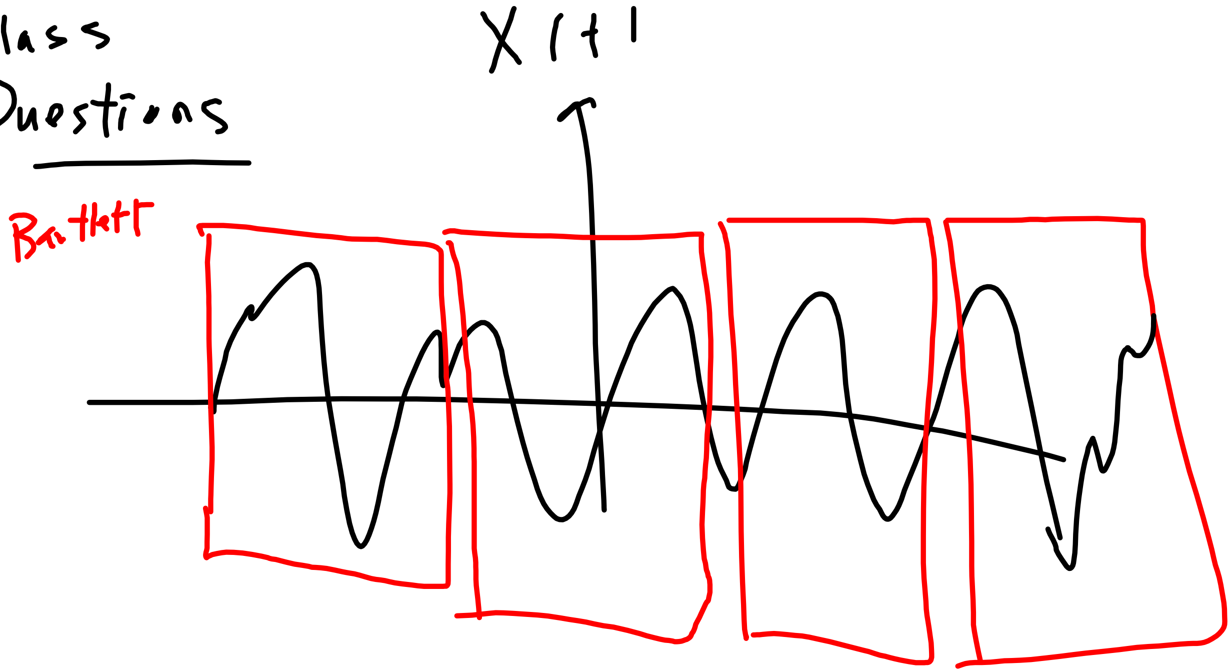
divide by "T" (or # of time samples);

$$\mathcal{E}\left\{\underbrace{\frac{1}{T}\left|\tilde{X}_T(f)\right|^2}_{\text{"spectrogram"}}\right\} = S(f)$$

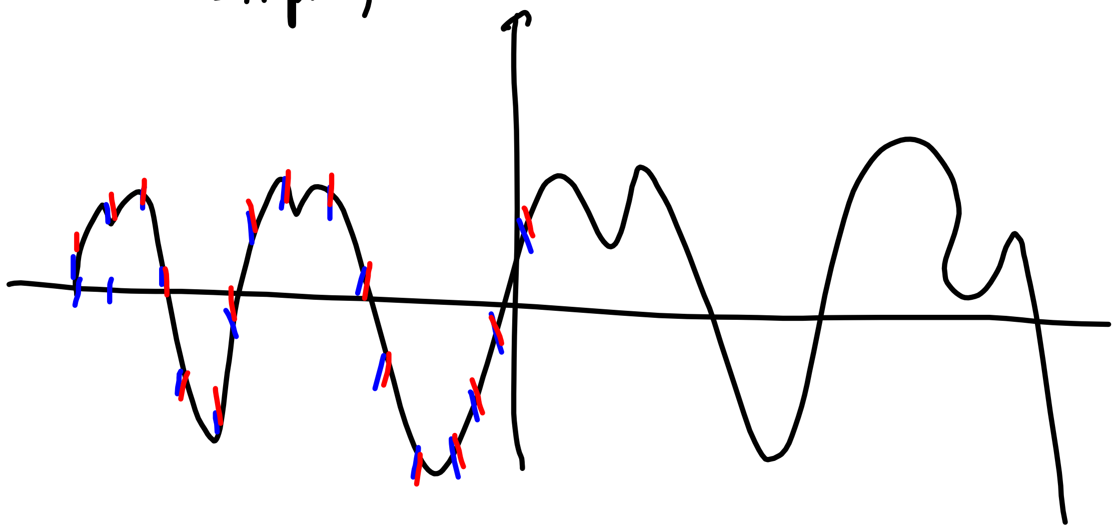
$$\mathcal{E}\left\{\text{inv. four. trans of } \frac{1}{T}\left|\tilde{X}_T(f)\right|^2\right\} = R_X(\tau)$$

"Bartlett's Method"

Class Questions



Suggested: sub sampling



Homework: run specestdemo.

When it asks for b, input [#]

$-1 < \# < 1$

(try 2).

When it asks for a,

input [1]

Return, return, ...