Notes for midterm.

①

Question: if $\ddot{X} + 2\dot{X} + 3X = V(t)$

& V is white noise with power $= \sigma^2 = 5$,

what is PSD of $x(t)$?

PSD of $V(t)$

Answer

$$PSD_X(f) = |H(\omega)|^2 \cdot 5$$

$$H(\omega) = \frac{1}{-\omega^2 + 2i\omega + 3}$$

Question: if $x(n) = 4v(n) - .2x(n-1) - .3x(n-2)$

and $v(n)$ is white noise, power $= 5$,

what is PSD of $x(n)$?

Ans. $$PSD_x = |H(\omega)|^2 \cdot 5$$

$$H(\omega) = \frac{4}{1 + .2e^{-i\omega} + .3e^{-i2\omega}}$$

Question: Stationary gaussian process

$x(t)$

Joint Gaussian processes

joint pdf $= \frac{1}{\sim} e^{\sim\sim}$ $\qquad \sigma_1, \sigma_2, \mu_1, \mu_2, \rho$

$$\text{Prob}(11 < x_1 < 12,\ 14 \le x_2 \le 16) = \int_{x_1=11}^{12} \int_{x_2=14}^{16} \text{joint}\, dx_1\, dx_2$$

$$\text{marginal prob}(14 \le x_2 \le 16) = \int_{14}^{16} \text{marginal}\, dx_2$$

$\rightarrow$ 1-D Gaussian, μ_2, σ_2.

$$\text{prob.}(12 \le x_1 \le 13 \text{ given } y = 7) = \int_{12}^{13} \text{conditional}\, dx$$

$\rightarrow$ mean, st dev $\}$ ..... μ's, σ's, ρ, "7"

$\text{pdf}(X_1^2)$

$y = x_1^2$

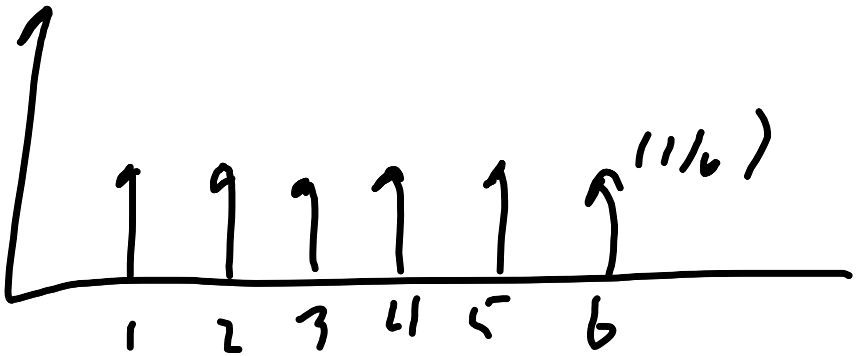
$x_1 = \sqrt{y}$

$x_1 = -\sqrt{y}$

$$f_Y(y)\,|dy| = f_{X_1}(x_1)\,|dx_1| + f_{X_1}(x_1)\,|dx_1|$$

$$f_Y(y) = \frac{f_{X_1}(x_1)}{|dy/dx_1|} + (\text{other one})$$

$y = x_1 + x_2$



$$\text{prob}(\text{sum}=3) = \text{prob}(1+2) + \text{prob}(2+1)$$

Think about: if $y = x(t_1) + x(t_2)$, (Gaussian)

what is pdf of y?

Question given $R_X(\tau)$ for a zero-mean Gaussian process, answer all the previous questions,

joint pdf $X(t_1)$, $X(t_2)$?

marginal $X(t_1)$?

conditional $X(t_1)$ given $X(t_2)$?

Question ARMA

both ways: given a's, b's
what are R_X's?

given $R_X(\tau)$, ~~to~~ given p, q,
what are a's & b's?

Questions

Write a paragraph describing the implementation of Bartlett's method.

Question: identify "ergodic".

Example of a stationary process that is not ergodic.

Flip a coin:								
H:	1	1	1	1	1	1	1	1
T:	0	0	0	0	0	0	0	0

Think about this:

$$x(n) = 4\,v(n) - .2\,x(n-1) - .3\,x(n-2)$$

PSD of $x(t)$

method 1. $PSD_x = |H(\omega)|^2 PSD_v$

method 2. $PSD_x = $ F.T. of $R_x(n)$, use Y-W to get $R_x(n)$

Why is $PSD_X = |H(\omega)|^2 PSD_V$?

$2\pi f$

<u>Def'n of transfer fcn.</u> $H(\omega)e^{i\omega t}$ is the solution when input is $e^{i\omega t}$.

Fact 1. (Lemma 1.)

$H(\omega)$, as defined, turns out to be the Four Trans of the response to an impulse:

$$\ddot{x} + 2\dot{x} + 3x = f(t)$$

transfer fcn. $\frac{d^2}{dt^2}\left(H(\omega)e^{i\omega t}\right) + 2\frac{d}{dt}\left(H(\omega)e^{i\omega t}\right) + 3\left(H(\omega)e^{i\omega t}\right) = e^{i\omega t}$

$x_{impulse}(t)$ $\quad \ddot{x}_{imp}(t) + 2\dot{x}_{imp}(t) + 3x_{imp}(t) = \delta(t)$

Why? Four Trans of impulse:

$$\cancel{\text{FT: } \hat{f}(f) = \int_{-\infty}^{\infty} f(t)\, e^{-i2\pi f t}\, dt}$$

$$\hat{y}(f) = \int_{-\infty}^{\infty} y(t)\, e^{-i2\pi f t}\, dt$$

$$1 = \hat{\delta}(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-i2\pi f t}\, dt$$