~~To do~~

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# 4 :

To estimate $x(n+2)$,

$$err = x(n+2) - w(0)x(n) - w(1)x(n-1) - w(2)x(n-2)$$

$$E[err^2] = \underbrace{x(n+2)^2}_{r_x(0)} + w(0)^2 \underbrace{x(n)^2}_{r_x(0)} + w(1)^2 \underbrace{x(n-1)^2}_{r_x(0)} + w(2)^2 \underbrace{x(n-2)^2}_{r_x(0)}$$

$$- 2\underbrace{x(n+2)w(0)x(n)}_{r_x(2)} - 2\underbrace{x(n+2)w(1)x(n-1)}_{r_x(3)}$$

$$\underbrace{-2x(n+2)w(2)x(n-2)}_{r_x(4)} + 2w(0)w(1)\underbrace{x(n)x(n-1)}_{r_x(1)}$$

$$+ 2w(0)w(2)\underbrace{x(n)x(n-2)}_{r_x(2)} + 2w(1)w(2)\underbrace{x(n-1)x(n-2)}_{r_x(1)}$$

$$\frac{\partial E[err^2]}{\partial w(0)} = 0 = 2w(0)r_x(0) - 2r_x(2) + 2w(1)r_x(1) + 2w(2)r_x(2)$$

$$\begin{bmatrix} r_x(0) & r_x(1) & r_x(2) \\ & \text{---} & \\ & \text{---} & \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ w(2) \end{bmatrix} = \begin{bmatrix} r_x(2) \\ \text{---} \\ \text{---} \end{bmatrix}$$

$$r_x(k) = (-1)^k / (|k| + 1)$$

$$\begin{bmatrix} 1 & -1/2 & 1/3 \\ & \text{---} & \\ & \text{---} & \end{bmatrix} \begin{bmatrix} w \\ w \\ w \end{bmatrix} = \begin{bmatrix} 1/3 \\ \text{---} \\ \text{---} \end{bmatrix}$$

To estimate $d(n)$:

$$Err = -d(n) + w(0)\,x(n) + w(1)\,x(n-1) + w(2)\,x(n-2))$$

$$\left[Err^2\right] = \underbrace{d(n)^2}_{r_d(0)} + w(0)^2\underbrace{x(n)^2}_{r_x(0)} + w(1)^2\underbrace{x(n-1)^2}_{r_x(0)} + w(2)^2\underbrace{x(n-2)^2}_{r_x(0)}$$
$$-2\underbrace{d(n)\,w(0)\,x(n)}_{r_{dx}(0)} - 2\underbrace{d(n)\,w(1)\,x(n-1)}_{r_{dx}(1)} - 2\underbrace{d(n)\,w(2)\,x(n-2)}_{r_{dx}(2)})$$
$$+2\,w(0)\,w(1)\underbrace{x(n)\,x(n-1)}_{r_x(1)} + 2\,w(0)\,w(2)\underbrace{x(n)\,x(n-2)}_{r_x(2)})$$
$$+2\,w(1)\,w(2)\underbrace{x(n-1)\,x(n-2)}_{r_x(1)})$$

$$\frac{\partial}{\partial w(0)} = 0 = 2\,w(0)\,r_x(0) - 2\,r_{dx}(0) + 2\,w(1)\,r_x(1) + 2\,w(2)\,r_x(2)$$

$$\begin{bmatrix} r_x(0) & + r_x(1) & + r_x(2) \\ & & \\ & & \end{bmatrix}\begin{bmatrix} w(0) \\ w(1) \\ w(2) \end{bmatrix} = \begin{bmatrix} r_{dx}(0) \\ \\ \end{bmatrix}$$

$$r_x(n) = \frac{(-1)^n}{|n|+1}$$

$$x(n) = d(n) + v(n)$$

$$r_{dx}(0) \equiv \mathcal{E}\left[x(n)\,d(n)\right]$$
$$= \mathcal{E}\left[x(n)\left(x(n) - v(n)\right)\right]$$
$$= \mathcal{E}\left[x(n)^2\right] - \mathcal{E}\left[x(n)\,v(n)\right]$$
$$= r_x(0) - \mathcal{E}\left[\left(d(n)+v(n)\right)v(n)\right]$$
$$= r_x(0) - \mathcal{E}\left[d(n)\,v(n)\right]$$
$$- \underbrace{\mathcal{E}\left[v(n)^2\right]}_{\sigma_v^2}$$

$d$ & $v$ are uncorrelated

$$= r_x(0) - \mathcal{E}\left[d(n)\right]\overset{\text{''}0}{\mathcal{E}\left[v(n)\right]}$$
$$- \sigma_v^2$$

$$\cancel{\{w(1)\, r_x(1)\} + \{w(2)\, r_x(2)\}} \qquad 3$$

$$\begin{bmatrix} r_x(0) + r_x(1) + r_x(2) \\ \\ \\ \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ w(2) \end{bmatrix} = \begin{bmatrix} r_{dx}(0) \\ \\ \\ \end{bmatrix} = \begin{bmatrix} r_x(0) - \sigma_v^2 \\ = \\ = \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/2 & 1/3 \\ \hline \\ \hline \end{bmatrix} \begin{bmatrix} w \\ w \\ w \end{bmatrix} = \begin{bmatrix} 1 - 3 \\ \\ \\ \end{bmatrix} = \begin{bmatrix} -2 \\ \\ \\ \end{bmatrix}$$

$$r_x(n) = \frac{(-1)^n}{|n| + 1}$$

We will need $r_{dx}(1) = E[d(n)\,x(n-1)]$

$$E[d(n)\,x(n-1)] = E\{[x(n) - v(n)]\,x(n-1)\}$$

$$= \underbrace{E[x(n)\,x(n-1)]}_{r_x(1) = -1/2} - \underbrace{E[v(n)\,x(n-1)]}_{E[v(n)\,(d(n-1) + v(n-1))]}$$

$$E[v(n)\,(d(n-1) + v(n-1))] = \underbrace{E[v(n)\,d(n-1)]}_{0} + \underbrace{E[v(n)\,v(n-1)]}_{0}$$