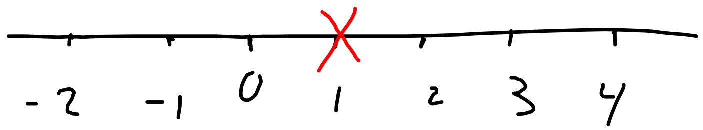
Kalman Example

at n=0



no uncertainty

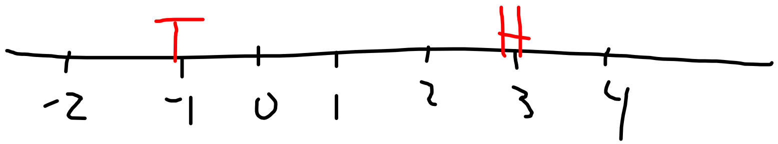
$\mathcal{E}[e_0^2] = 0$

$\hat{d}(0) = 1$

$a(n-1)$

$d(1) = 1 \cdot d(0) + v_1(1)$

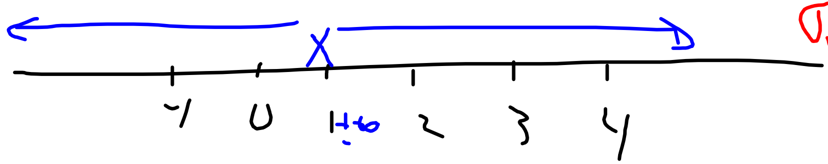
flip a coin!



$v_1(1) = \pm 2$

$\sigma_{v_1}^2 = 4$

measure ⟹



What is Kalman estimate of position?

$$P_0 = a(n-1)^2 \mathcal{E}[e_0^2] + \sigma_{v_1}^2 = 1 \cdot 0 + 4 = 4$$

$$K(1) = \frac{4}{4+\infty} = \frac{P_0}{P_0 + \sigma_{v_2}^2} = 0$$

$$\hat{d}(1) = a(n-1)\hat{d}(0) + K(1)\left[x(1) - a(n-1)\hat{d}(0)\right]$$

$$= 1 \cdot 1 + 0\,[1 - 1 \cdot 1] = 1$$

Summary: The marker is either at -1 (50%) or $+3$ (50%).

Kalman says, estimate it to be at $+1$.
(Wrong)

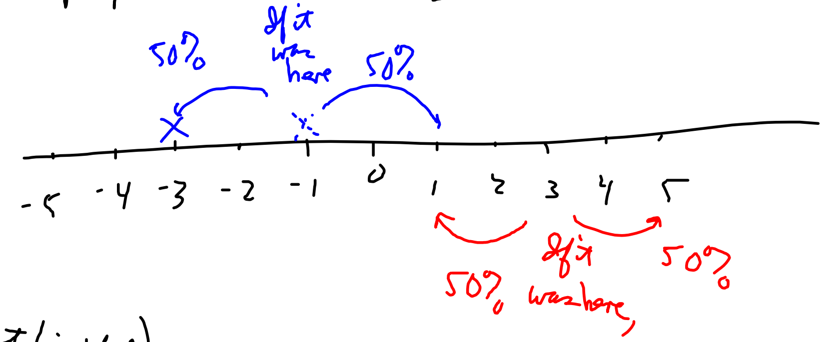
$$MSE = \frac{1}{2}\left[1-(-1)\right]^2 + \frac{1}{2}\left[1-3\right]^2$$

$$= 4$$

What if you estimated it to be at -1? (Right 50%)

$$MSE = \frac{1}{2}\left[-1-(-1)\right]^2 + \frac{1}{2}\left[-1-3\right]^2 = 8$$

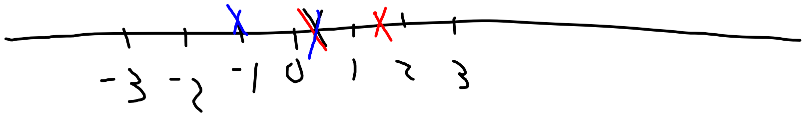
2nd flip: move marker ± 2



(not finished)

$$d_{prelim}(2) = d(1) \pm 2$$

Take $1/3$



$$d(2) = \frac{1}{3} d_{prelim}(2) = \frac{1}{3} d(1) \pm \frac{2}{3}$$

$a(n-1)$ $\qquad v_1$ $\qquad \sigma_{v_1}^2 = 4/9$

measure $x(2) = d(2) + v_2$

$x(2) = 0.5$ $\qquad \sigma_{v_2} = 1$

What is $\mathcal{E}[e_1^2] = \frac{P_0 \sigma_{v_2}^2}{P_0 + \sigma_{v_2}^2} = \frac{P_0(\infty)}{P_0 + \infty}$

$$= \frac{P_0}{\frac{P_0}{\sigma_{v_2}^2} + 1} = \frac{P_0}{0 + 1} = P_0 = 4 \text{ (page 1)}$$

$a(n-1) = 1/3 \qquad \hat{d}(1) = 1$

So $P(1) = \left(\frac{1}{3}\right)^2 \cdot 4 + \frac{4}{9} = 8/9$

$$K(2) = \frac{P(1)}{P(1) + \sigma_{v_2}^2} = \frac{8/9}{8/9 + 1^2} = \frac{8}{17}$$

$$\hat{d}(2) = a(n-1)\hat{d}(1) + K(2)\left[x(2) - a(n-1)\hat{d}(1)\right]$$

$$= \frac{1}{3} \cdot 1 + \frac{8}{17}\left[.5 - \frac{1}{3} \cdot 1\right]$$

$$= \frac{1}{3} + \frac{8}{17} \cdot \frac{1}{6} = \frac{1}{3} + \frac{4}{51} = \frac{21}{51} \text{ (???)}$$

$$= \frac{1}{3} + \frac{4}{51} \;\;(\text{"} = 17\text{"}) \qquad = 7/17.$$

From the diagram, we know $d(2)$ is either -1 (25%), $1/3$ (50%), or $5/3$ (25%).

But the measurement was $.5$ with some noise ($\sigma_{v_2}^2 = 1$).

So ~~estimate~~ without the measurement we would guess $1/3$.

With the measurement, move this up to ~~[illegible]~~ $7/17$.