Well known fact about Wiener ~~filter~~ filter

Use $[x(n) \;\; x(n-1) \cdots x(n-k)] = x^T$

error (using Wiener coefficients)

$\mathcal{E}[\text{error}\ x(n)] = 0$ "orthogonal"

$\hat{d}(n) - d(n)$

$= \mathcal{E}[\text{error}\ x(n-1)]$

$= \mathcal{E}[\text{error}\ x(n-2)]$

$\mathcal{E}[\text{error}\ x] = [0]$

$$\text{error} = d(n) - \hat{d}(n)$$

$$= d(n) - \underbrace{\sum x(n-k)\,w(k)}_{x^T w} \quad \leftarrow \text{scalar}$$

$$\begin{bmatrix} x \\ \\ \\ \end{bmatrix} \text{error} = x\,d(n) - x\,x^T w \rightarrow r_d^{-1} r_{dx}$$

$$\mathcal{E}[x \,\text{error}] = \underbrace{\mathcal{E}[x\,d]}_{dx} - \underbrace{\mathcal{E}[x\,x^T]}_{r_d} r_d^{-1} r_{dx}$$

$$= r_{dx} - r_d^{\,T}\, r_d^{-1}\, r_{dx} = 0$$

Fairly well known fact

$$\mathcal{E}[\text{error}^2] = r_d(0) - r_{dx}^T w$$

$$= r_d(0) - r_{dx}(0)\,w(0) - r_{dx}(1)\,w(1)\ldots$$

Why?

$$\mathcal{E}[\text{err}^2] = \mathcal{E}\left[(d-\hat{d})^2\right]$$

$$= \mathcal{E}\left[(d - x^T w\ )^2\right] \qquad w = r_d^{-1} r_{dx}$$

$$= \mathcal{E}\Big[\big(d - \underbrace{x^T r_d^{-1} r_{dx}}_{r_{dx}^T\, r_d^{-1T}\, x}\big)\big(d - x^T r_d^{-1} r_{dx}\big)\Big]$$

$$= \mathcal{E}[d^2] - r_{dx}^T r_d^{-1}\,\mathcal{E}[x\,d] - \mathcal{E}[d\,x^T]\,r_d^{-1}\,r_{dx}$$

$$+ r_{dx}^T r_d^{-1}\,\mathcal{E}[x\,x^T]\,r_d^{-1} r_{dx}$$

$$= \mathcal{E}[d^2] - r_{dx}^T r_d^{-1} r_{dx} - r_{dx}^T r_d^{-1} r_{dx}$$

$$+ r_{dx}^T \underbrace{r_d^{-1} r_d}\, r_d^{-1} r_{dx}$$

$$= \mathcal{E}[d^2] - r_{dx}^T r_d^{-1} r_{dx}$$

$$\mathcal{E}[\text{err}^2] = r_d(0) - r_{dx}^T\, w$$

Problem 1, 12/3/03 final.

After first flip, $x(0) = \pm 1$ don't know

$$\hat{d}(0) = 0$$

$$\mathcal{E}(e_0^2) = (+1)^2 \times \tfrac{1}{2} + (-1)^2 \times \tfrac{1}{2} = 1$$

$$d(n) = \frac{d(n-1) + v_1(n)}{3}$$

$$d(n) = \underset{1/3}{a(n-1)}\, d(n-1) + v_1(n)$$

$\frac{1}{3}$ of coin-flip v_1

$\sigma_{v_1} = \frac{1}{3}\sigma_{v_1} = \frac{1}{3}$

$\sigma_{v_1}^2 = 1/9$

$$x(n) = d(n) + v_2(n)$$

$\sigma_{v_2}^2 = 1^2 = 1$

$$\hat{d}(n=1) = \underbrace{a(n-1)}_{1/3}\,\underbrace{\hat{d}(n-1)}_{0} + \underbrace{K(n)}_{?}\left[\underbrace{x(n)}_{0.5} - \underbrace{a(n-1)}_{1/3}\,\underbrace{\hat{d}(n-1)}_{0}\right]$$

$$= K(n) \times 0.5$$

$$K(n) = \frac{a(n-1)^2\, E\left[e_{n-1}^2\right] + \sigma_{v_1}^2}{\left[\text{—— same ——}\right] + \sigma_{v_2}^2}$$

$$= \frac{\left(\frac{1}{3}\right)^2 (1) + \frac{1}{9}}{\left[\frac{2}{9}\right] + 1} = \frac{2/9}{2/9 + 1} = \frac{2}{11}$$

$$\hat{d}(1) = \frac{2}{11} \cdot (0.5) = \boxed{\frac{1}{11}}$$