Cut : guess 2 cards

2 ♢, 8 ♤

Prob?

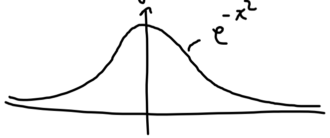
~~P~~ Prob(1st guess right & 2nd guess right)

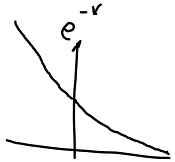
$$= \underbrace{\text{Prob}(\text{1st guess})}_{1/52} \; \underbrace{P(\text{2nd guess} \mid \text{1st guess})}_{\frac{1}{51}}$$

given

Gaussian Distribution, "Normal distribution"

Bell-shaped curve





Consider e^{-x^2} as a candidate for a pdf.

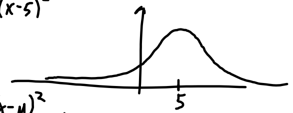
① $\int_{-\infty}^{\infty} e^{-x^2}\,dx = 1$ is false

So must use const e^{-x^2}. (Renormalize it)

② mean of x = $E(x)$ = const $\int_{-\infty}^{\infty} x\, e^{-x^2}\,dx = 0$

Shift the curve so that the mean is at $E(x) = 5$

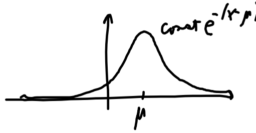
pdf = const $e^{-(x-5)^2}$

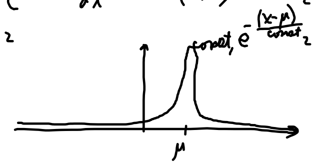


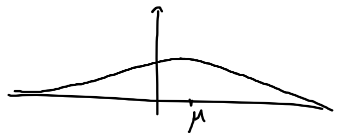
Generic pdf = const $e^{-(x-\mu)^2}$ has mean = μ.

③ Standard deviation = $\sqrt{E\left((x-\mu)^2\right)}$

$$= \sqrt{\text{const}_1 \int_{-\infty}^{\infty} (x-\mu)^2 e^{-(x-\mu)^2}\,dx} = (\#)$$







To get a s.d. of 10, use

$$\frac{1}{\sqrt{2\pi}\;10}\, e^{-\frac{(x-\mu)^2}{2\cdot 10^2}}$$

To get a s.d. of σ: pdf = $\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Summary The Gaussian (normal) distribution with mean $= \mu$, st. dev. $= \sigma$, has pdf

$$N(\mu,\sigma) \equiv \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

① $$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

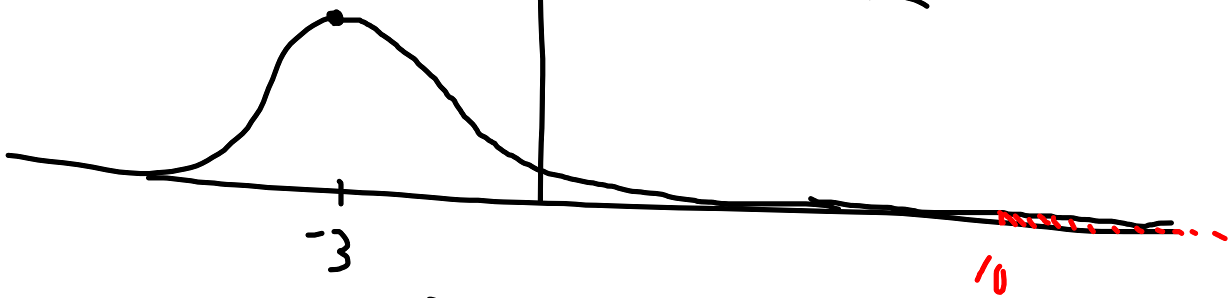
② $$\mathcal{E}(x) = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = \mu$$

③ $$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

Example: X is $N(-3,2)$

what is prob. $X > 10$?

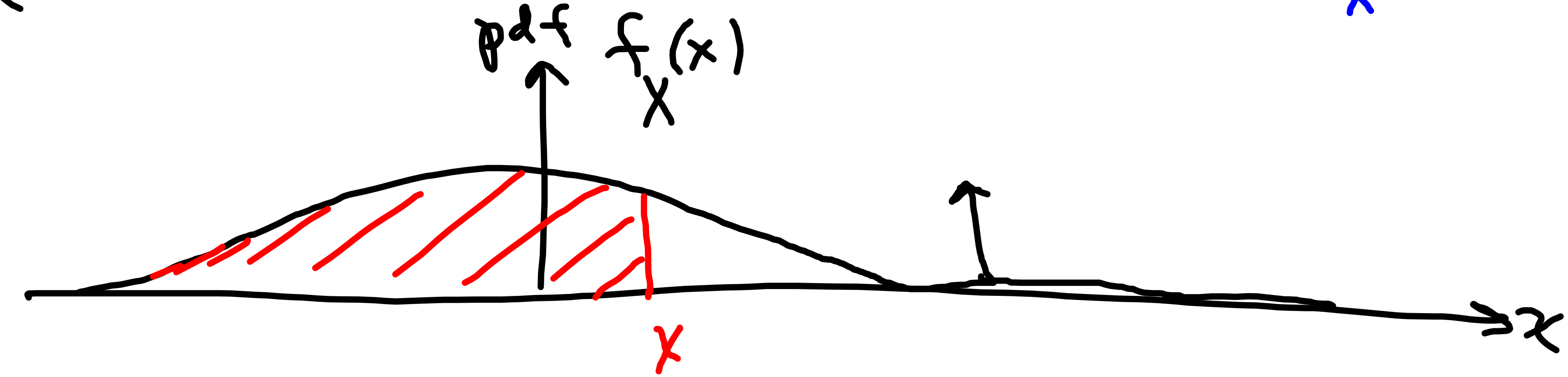
Answer: pdf $= N(-3,2) = \frac{1}{\sqrt{\pi}\,2} e^{-\frac{(x+3)^2}{2\cdot 2^2}}$



answer $= \int_{10}^{\infty} \frac{1}{\sqrt{\pi}\,2} e^{-\frac{(x+3)^2}{2\cdot 2^2}}\, dx$

(must be evaluated numerically)

Cumulative distribution function. = $F_X(x)$



$$F_X(x) = \text{Prob}(X \le x) = \int_{-\infty}^{x} \text{pdf}(x)\,dx$$

$$\frac{dF_X(x)}{dx} = f_X(x)$$

I do not use CDF.

X is $N(-3,2)$; what is Prob$(X>10)$?

Fact: If $\mathcal{E}(X) = \mu$ & st dev $= \sigma$,

① $\mathcal{E}(X-\mu) = 0$ why? $\mathcal{E}[X-\mu] = \mathcal{E}[X] - \mathcal{E}[\mu]$

$= \mu - \mu$

② st dev $\left(\frac{X-\mu}{\sigma}\right) = 1$

why?

$$\text{st dev}^2 = \mathcal{E}\left[\left(\frac{X-\mu}{\sigma} - \mathcal{E}\left(\frac{X-\mu}{\sigma}\right)\right)^2\right]$$

homework: carry this out.

So: if you subtract the mean from the random variable, the result is zero mean.

if you rescale the random variable by σ, the result has unit st. dev.

If X is $N(-3, 2)$, then $\frac{X+3}{2}$ has zero mean, unit st dev.

$\Rightarrow$ Also, $\frac{X+3}{2}$ has Gaussian distribution!!

(Not true for all pdf's.)

Try this. X is $N(-3, 2)$

$$Y = \frac{X+3}{2}$$

What is pdf for Y? mean of $Y = 0$ st dev $Y = 1$

Y is $N(0, 1)$ "standard normal distribution"

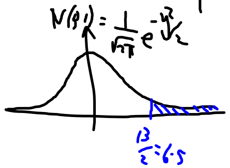
$$\text{pdf}(y) = f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

Back X is $N(-3,2)$

what is prob. $(X>10)$?

$\Longrightarrow$ Reduce to a ~~nor~~ standard normal R.V.,

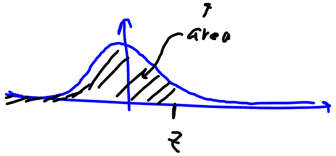
$$Y = \frac{X+3}{2}$$



$$\text{Prob}(X>10) = \text{Prob}\left(Y > \frac{10+3}{2}\right)$$

$$\frac{10+3}{2} = 13/2$$

On p. 123, $\Phi(z)$ is tabulated.



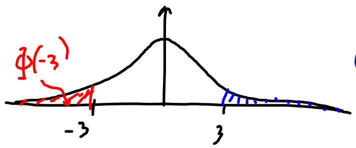
$\Phi(0) = .5$

$$\text{Prob}\left(Y > \frac{13}{2}\right) = \text{area to right of } 13/2$$

$$= 1 - \Phi\left(\frac{13}{2}\right)$$

Note:

$\Phi(-3)$ in terms of $\Phi(3)$



$\Phi(-3)$ is the same as the area to the right of $+3$, by symmetry.

So $\Phi(-3) = 1 - \Phi(3)$

(Φ is only tabulated for $z>0$).

$\Phi\left(\frac{13}{2}\right) = ?$ The book stops at $\Phi(2.99) = .99861$

Try this.

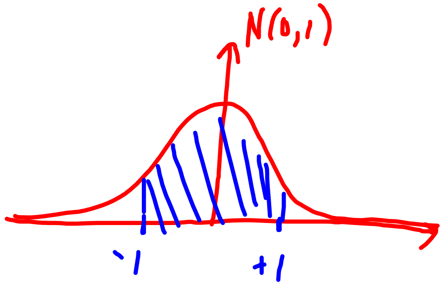
X is $N(4, 3)$

What is $\text{Prob}(1 < X < 7)$?

Define $Y = \dfrac{X-4}{3}$ $\quad Y$ is $N(0,1)$

$$\text{Prob}(1 < X < 7) = \text{Prob}\left(\frac{1-4}{3} < Y < \frac{7-4}{3}\right)$$

$$= \text{Prob}(-1 < Y < 1)$$



$$= \Phi(1) - \Phi(-1)$$

$$\quad\quad = 1 - \Phi(1)$$

$$= 2\Phi(1) - 1$$

$$= 2\cdot(.8413) - 1 = .6826$$