$$\vec{\nabla} = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

$\vec{\nabla}\phi$ "gradient" normals to isotherms, direction of maximum change.

$\vec{\nabla}\cdot\vec{A}$ "divergence" : outflux per unit volume

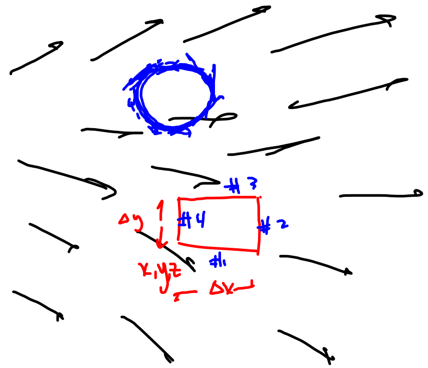
$\vec{\nabla}\times\vec{A}$ "curl" : swirl, vortex, $\frac{\text{angular velocity}}{2}$

$\vec{\nabla}\cdot\vec{\nabla}\phi$ "Laplacian"

$\vec{\nabla}\times[\vec{\nabla}\phi]$ always zero.

$\vec{\nabla}\times[\vec{\nabla}\times\vec{F}]$ ugly.

Interpretation of $\nabla \times \vec{F}$:



To what extent does the vector field $\vec{F}$ drive the circulation?

	#1	#2	#3	#4
	$F_1 \Delta x$	$F_2 \Delta y$	$-F_1 \Delta x$	$-F_2 \Delta y$
	at y	at $x+\Delta x$	at $y+\Delta y$	at x

$$-\frac{F_1(y+\Delta y)-F_1(y)}{\Delta y}\,\Delta x\,\Delta y+\frac{F_2(x+\Delta x)-F_2(x)}{\Delta x}\,\Delta x\,\Delta y$$

$$\left[-\frac{\partial F_1}{\partial y}+\frac{\partial F_2}{\partial x}\right]\Delta x\,\Delta y$$

Define curl to be circulation per unit area:

$$\mathrm{curl}_z = -\frac{\partial F_1}{\partial y}+\frac{\partial F_2}{\partial x}$$!!!

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \vec{i}\left(\quad\right) + \vec{k}\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right) + \vec{j}\left(\frac{\partial}{\partial z}F_1-\frac{\partial}{\partial x}F_3\right)$$

Example 3.18 $\overrightarrow{\text{curl}}\left(xyz\vec{i} + x^2y^2z^2\vec{j} + y^2z^3\vec{k}\right)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & x^2y^2z^2 & y^2z^2 \end{vmatrix}$$

$$= \vec{i}\left(2yz^2 - 2x^2y^2z\right) + \vec{j} \ \_\_\_\_\_\_$$

$$+ \vec{k} \ \_\_\_\_\_\_$$

Example $\vec{\nabla} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

$$= \vec{0}$$

Example $\vec{\omega} \times \vec{R}$

$$\vec{\omega} = \omega_1 \vec{i} + \omega_2 \vec{j} + \omega_3 \vec{k}$$

$$\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{\omega} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_1 & \omega_2 & \omega_3 \\ x & y & z \end{vmatrix} = \vec{i}\left(\omega_2 z - \omega_3 y\right) + \vec{j}\left(\quad\right) + \vec{k}\left(\quad\right)$$

$$\vec{\nabla} \times [\vec{\omega} \times \vec{R}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (\omega_2 z - \omega_3 y) & (\quad) & (\quad) \end{vmatrix} = 2\vec{\omega}$$

$$= (2\omega_1)\vec{i} + (2\omega_2)\vec{j} + (2\omega_3)\vec{k}$$

p.132 # 9

$\vec{\nabla} \times \vec{F} = y\vec{i}$ $\quad \vec{F} = ?$, $\frac{y^2}{2}\vec{k}$ or $-yz\vec{j}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ?, & ?, & ?, \end{vmatrix} = y\vec{i}$$

$0 \quad -yz \text{ or } y^2/2$

p. 132 # 9

$$\vec{\nabla} \times \vec{F} = x\,\vec{i} \qquad \vec{F} = ?$$

impossible

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ? & ? & ? \end{vmatrix} = x\,i$$

$$-xz \qquad xy$$

(Note: will explain later

$$\operatorname{div} x\vec{i} = \vec{\nabla} \cdot x\vec{i} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \cdot \left(x\vec{i}\right)$$

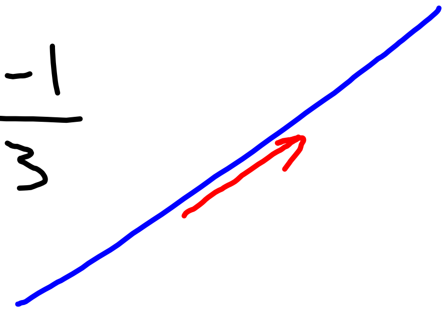
$$= \frac{\partial x}{\partial x} = 1$$

$$\nabla \cdot y\vec{i} = \frac{\partial y}{\partial x} = 0$$

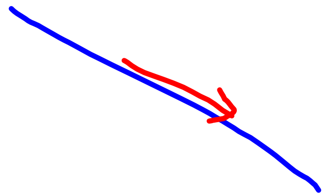
$$\vec{\nabla} \cdot y\vec{j} = 1$$

p. 53 #27

line #1 $\quad \frac{x}{2} = y = \frac{z-1}{3}$



line #2 $\quad \frac{x}{3} = y = z$



Step 1. Find a vector <u>along</u> each line.

Step 2. Cross these 2 vectors: the result will be $\perp$ to each line.

Step 3. Find a point on line 1 & a point on line 2 so that the vector between them is $\parallel$ to vector in step 2.

p. 53 #27

line #1 $\quad \dfrac{x-0}{2} = \dfrac{y-0}{1} = \dfrac{z-1}{3} \qquad \dfrac{x-x_0}{v_1} = \dfrac{y-y_0}{v_2} = \dfrac{z-z_0}{v_3}$

$\vec{v} = 2\vec{i} + \vec{j} + 3\vec{k}$ || to line 1

line #2 $\quad \dfrac{x-0}{3} = \dfrac{y-0}{1} = \dfrac{z-0}{1}$

$\vec{w} = 3\vec{i} + \vec{j} + \vec{k}$

is || to line 2.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 3 \\ 3 & 1 & 1 \end{vmatrix} = \vec{i}(-2) + \vec{j}(7) + \vec{k}(-1)$$

$$= \boxed{-2\vec{i} + 7\vec{j} - \vec{k}}$$

Here's what I'm going to do.

1. Write a generic formula for a point on line 1. $\vec{R}_1(t) = \underset{\text{on line}}{\vec{R}_0} + t\,\underset{\parallel \text{ to line}}{\vec{V}}$

$(0\vec{i} + 0\vec{j} + 1\vec{k}) + t(2\vec{i} + \vec{j} + 3\vec{k})$

2. .... line, $\vec{R}_2(s) = \vec{0} + s(3\vec{i} + \vec{j} + \vec{k})$

3. Enforce $[\vec{R}_1(t) - \vec{R}_2(s)] \parallel -2\vec{i} + 7\vec{j} - \vec{k}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t-3s & t-s & 1+3t-s \\ -2 & 7 & -1 \end{vmatrix} = \vec{0}$$

$\vec{i} \to 0 = (t-s)(-1) - (1+3t-s)(7)$

$\vec{j} = 0 = (1+3t-s)(-2) - (2t-3s)(-1)$

$\vec{k} = 0 = (2t-3s)(7) - (t-s)(-2)$

$\square s + \square t = \square$

$\square s + \square t = \square$

$\square s + \square t = \square$