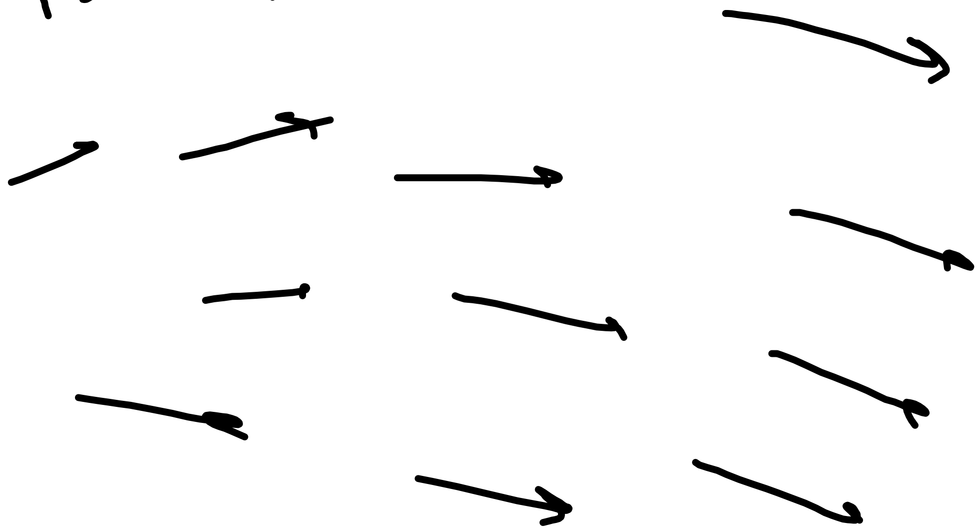
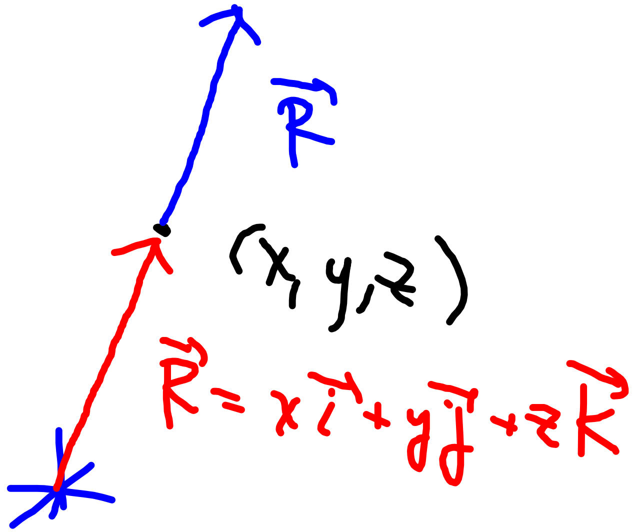
Vector fields.



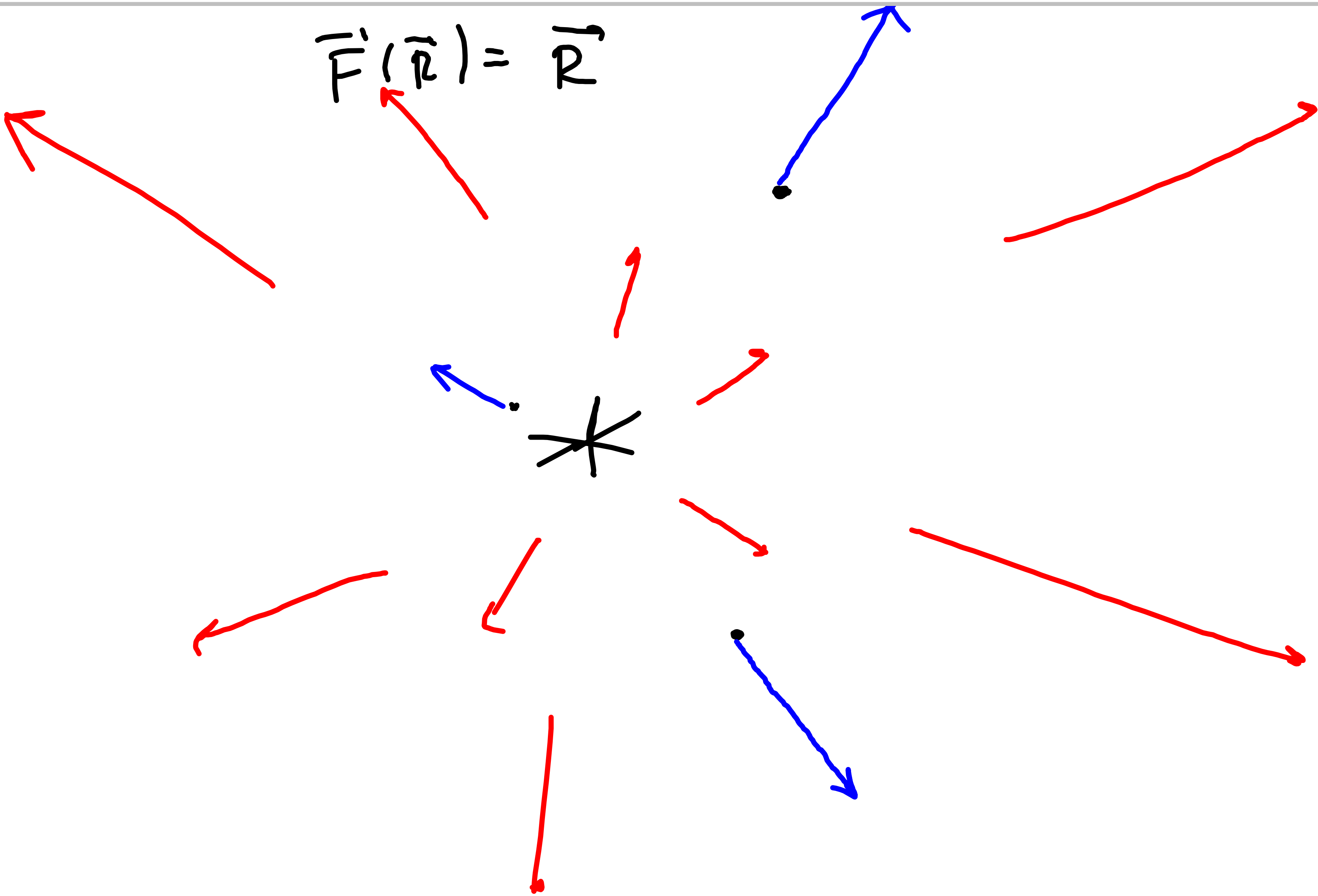
$$\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k} = \vec{F}(\vec{R})$$

$F_1(x, y, z)$

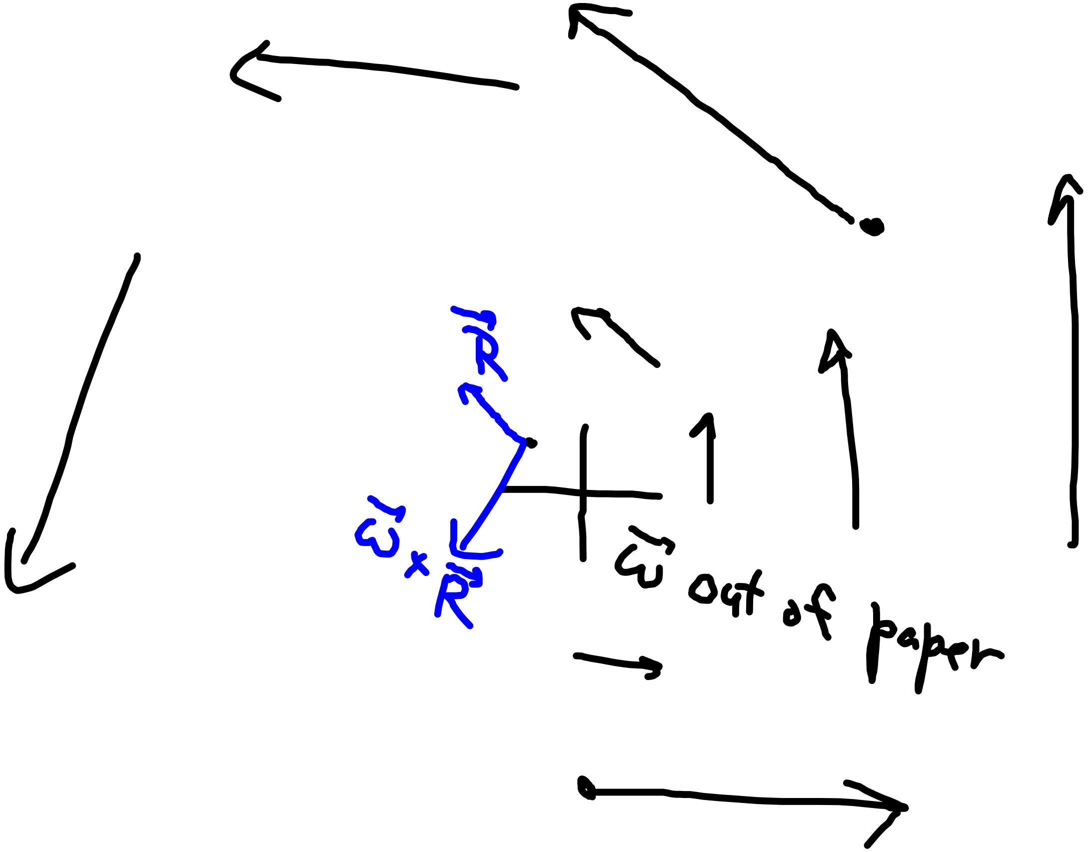
The vector field $\vec{F}(\vec{R}) = \vec{R}$



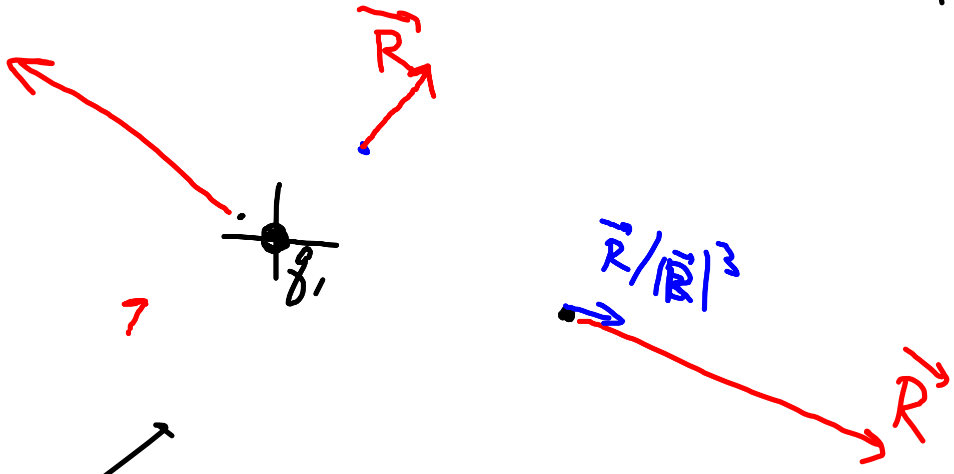
$$\vec{F}(\vec{R}) = \vec{R}$$

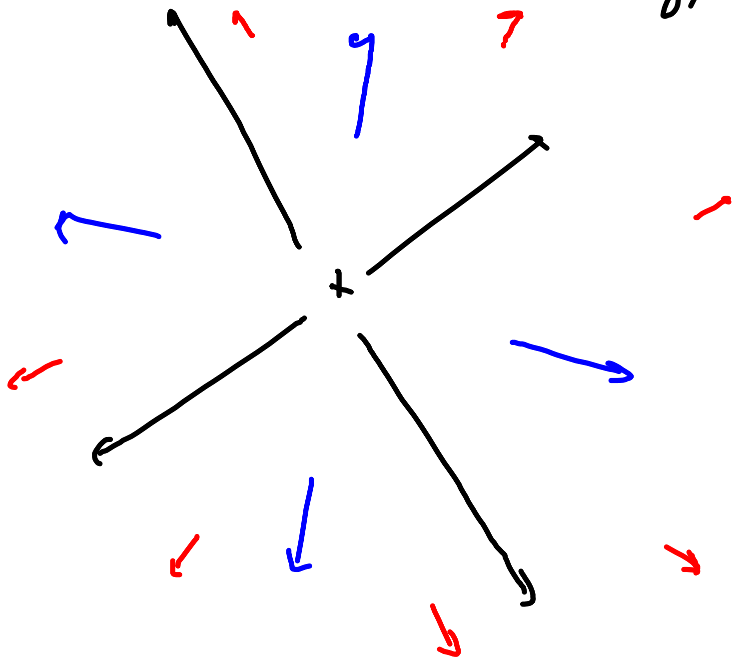


$$\vec{F} = \vec{\omega} \times \vec{R}$$



Coulomb field $\quad \vec{E} = (-)\,\dfrac{\vec{R} - \vec{O}}{|\vec{R} - \vec{O}|^3} = \dfrac{\vec{R}}{|\vec{R}|^3}$

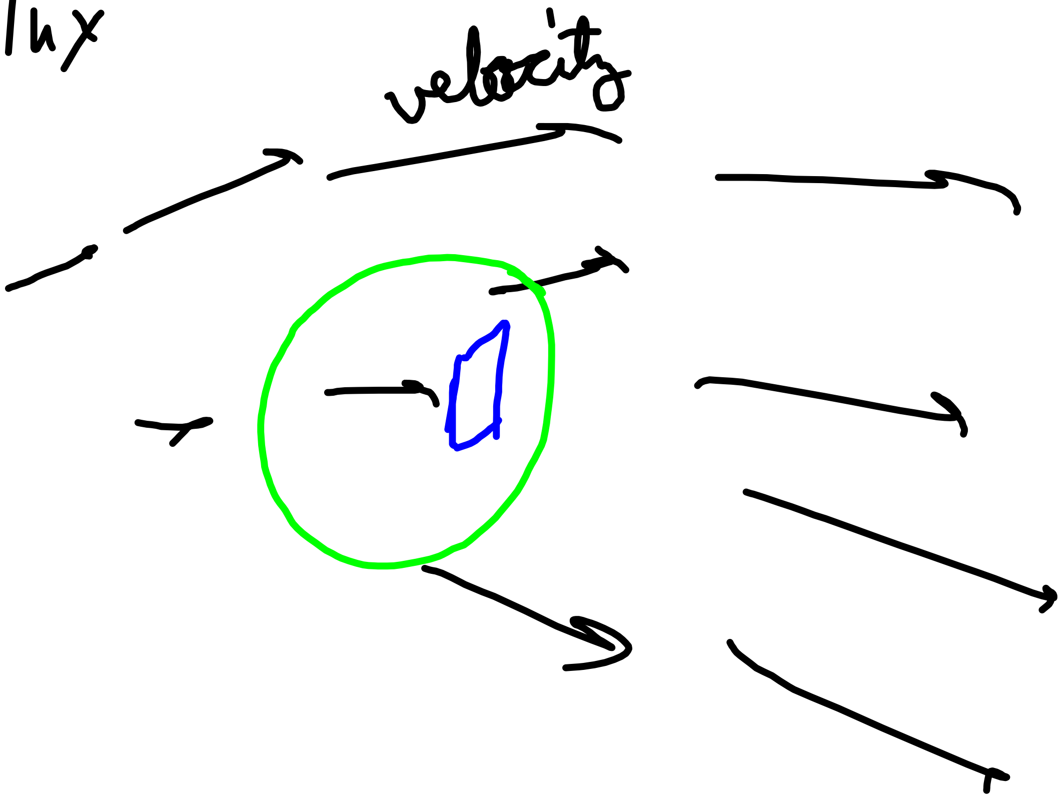




$\dfrac{\vec{R}}{|\vec{R}|^3}$, no curl,

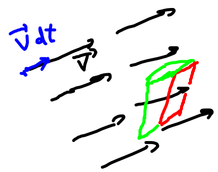
div ?

Flux



Microscopically

In time dt, the particle moves this far



The volume of the fluid that penetrates the window in time Δt = base x height

$$= \Delta S * (\vec{v}\,\Delta t) \cdot \vec{n}_{\text{unit normal}}$$

~~The flow rate~~

The # of particles in this volume

= particle density x volume

$= \nu \cdot (\text{vol})$

The mass that penetrates = $\underbrace{\text{mass density}}$ x volume

$$m_{\text{per particle}} \cdot \nu \cdot (\text{vol})$$

The charge that penetrates is charge density x (vol)

$$= e_{\text{charge per particle}} \cdot \nu \times (\text{vol})$$

$$= \rho \times (\text{vol})$$

ρ ← charge density

The amount of (whatever) penetrating in time

Δt = density of (") $\times \Delta S_{\text{area}} \; \vec{n}_{\text{unit}} \cdot \vec{v} \, \Delta t$

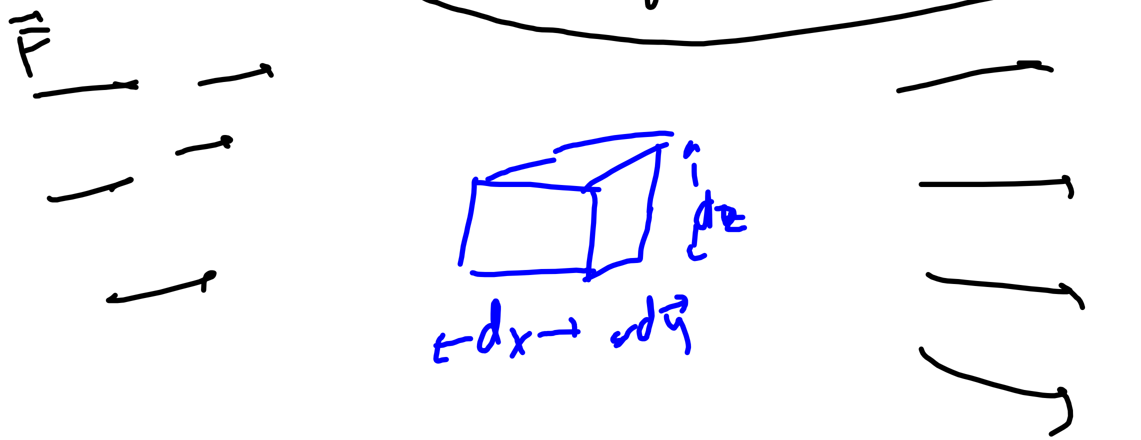
The <u>flux density</u>, per unit area, per unit time,

$$= \rho_{\text{whatever}} \vec{v} \cdot \vec{n}$$

Current density = $\vec{j} = \rho \vec{v}$.

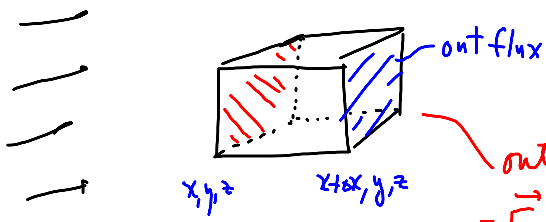
Divergence $\vec{\nabla}\cdot\vec{F} = \left(\vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} \cdots\right)\cdot\left(F_1\vec{i} + F_2\vec{j} + \cdots\right)$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$



Divergence will turn out to be

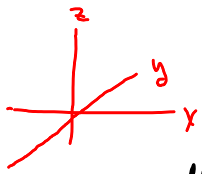
out flux per unit volume.



outflux

$= \vec{F} \cdot \vec{n}_{unit} \cdot \text{area}$

$= \underbrace{\vec{F} \cdot \vec{i}}_{F_1 \text{ at } x+\Delta x} (dy\, dz)$



outflux through the left face

$= \vec{F} \cdot (-\vec{i})\, dy\, dz = -F_1\, dy\, dz$ at x

The x-face contribution to the outflux is

$$\underbrace{\left[F_1(x+\Delta x, y, z) - F_1(x, y, z)\right]}_{\frac{\partial F_1}{\partial x}\Delta x} \Delta y\, \Delta z$$

Add in the y-face & z face contributions:

$$\text{net outflux} = \frac{\left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right)\Delta x\, \Delta y\, \Delta z}{\Delta x\, \Delta y\, \Delta z} = \vec{\nabla} \cdot \vec{F}$$

per unit volume

$$\vec{\nabla} \cdot \vec{R} = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z = 3$$

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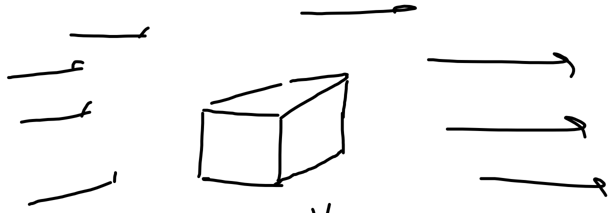
Conservation Laws.



volume V

density

$$\frac{d}{dt}\left(\text{amount of whatever inside the box}\right) = \frac{d}{dt}(\rho V)$$

" charge

$= \text{influx}$

$\left(-\text{outflux}\right)$

$\left(-\text{divergence of } \rho\vec{v} \times V\right) + \text{source}$

$$\frac{d}{dt}(\rho V) = -\vec{\nabla}\cdot(\rho\vec{v}V) + (\text{source})$$

$$\frac{\partial\rho}{\partial t} = \frac{d}{dt}\rho = -\vec{\nabla}(\rho\vec{v}) + \text{source}$$

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot\vec{j}$$

Conservation Law.

"Equation of continuity" (?)