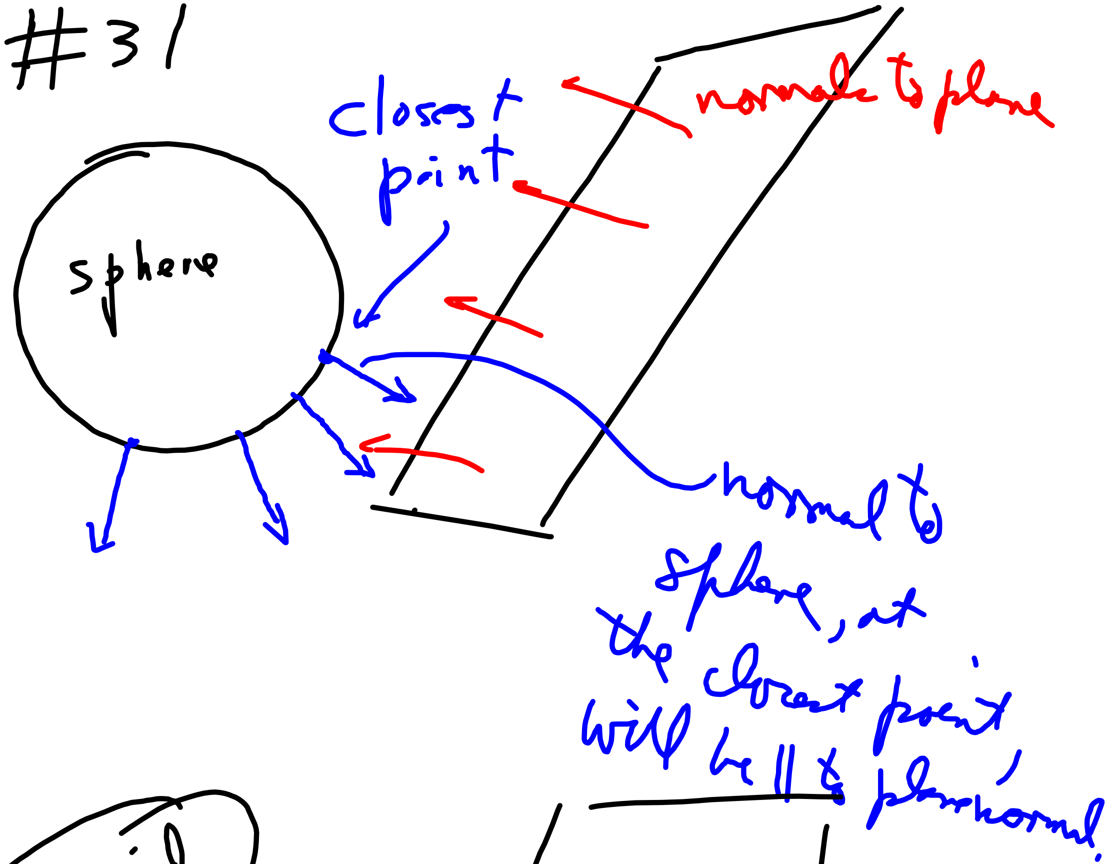
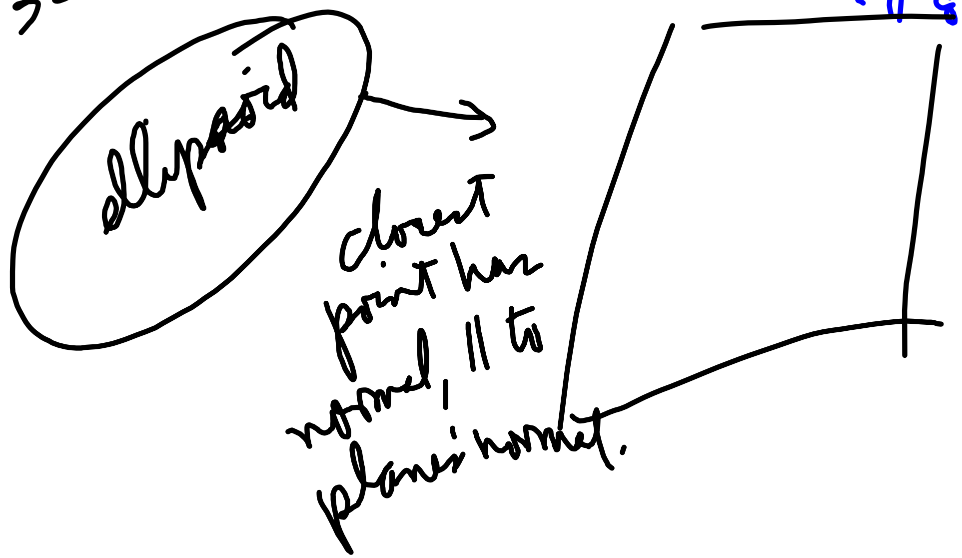
p. 114 #31


sphere
closest point
normals to plane
normal to sphere, at the closest point will be ∥ to plane normal.

#32


ellipsoid
closest point has normal ∥ to plane's normal.

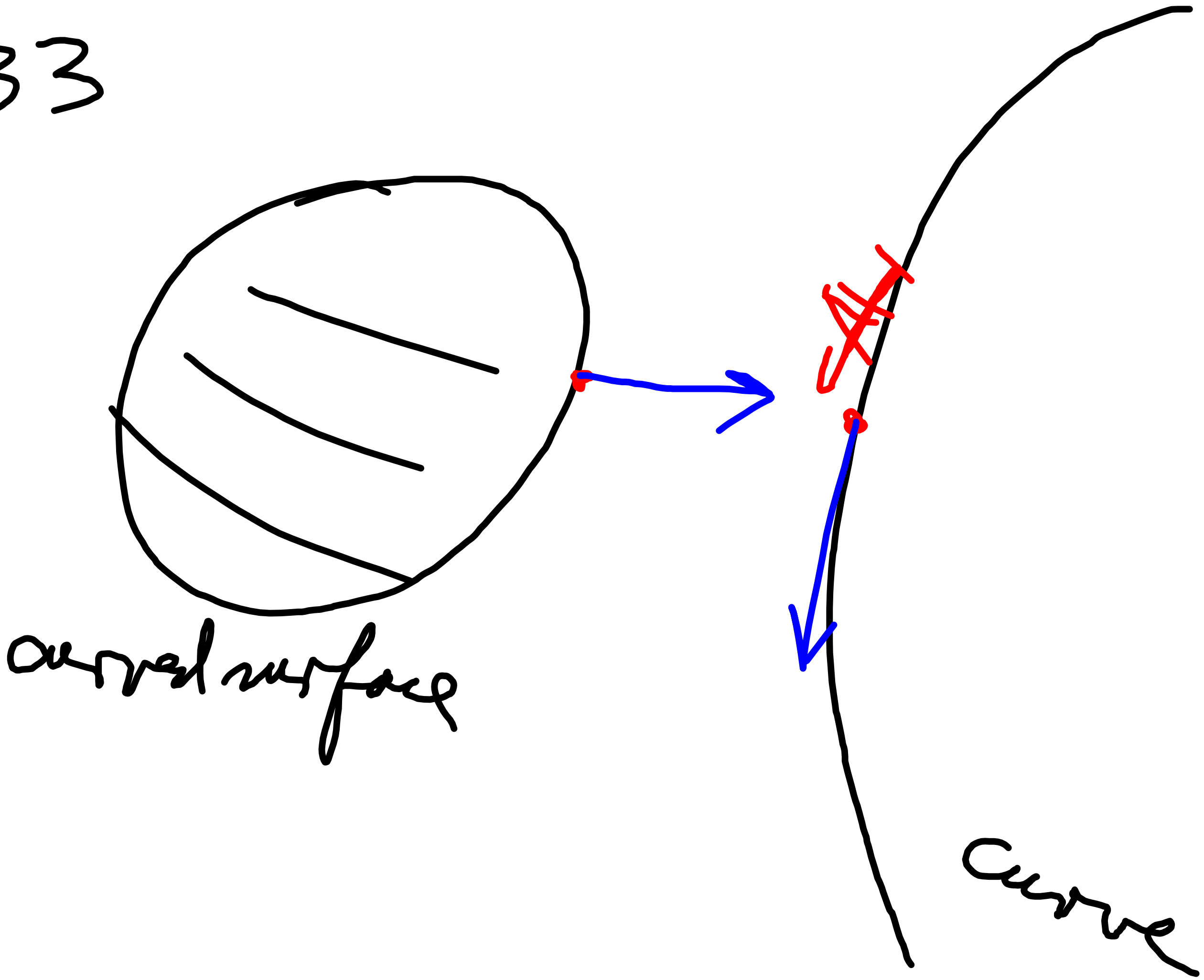
Normal to plane

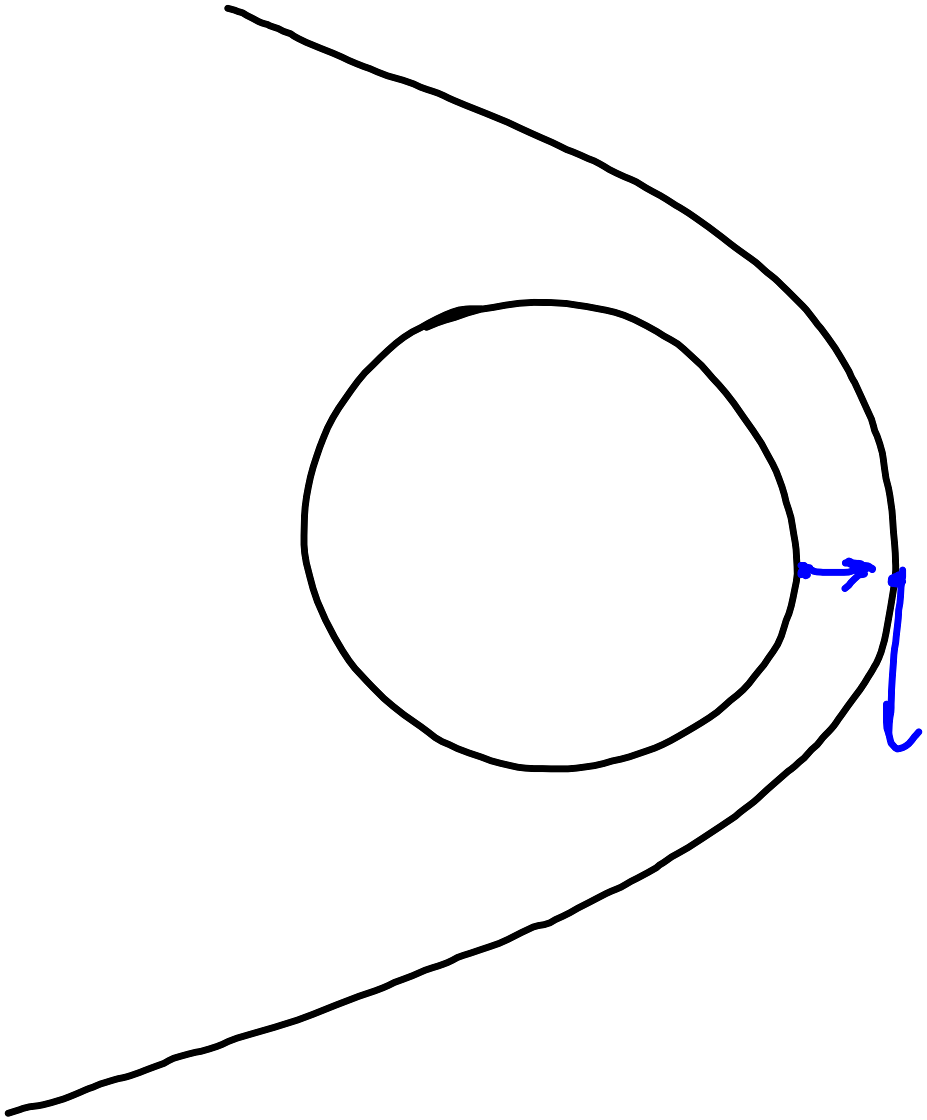
$$x + 2y + 3z = 8$$

① look it up on sheet.

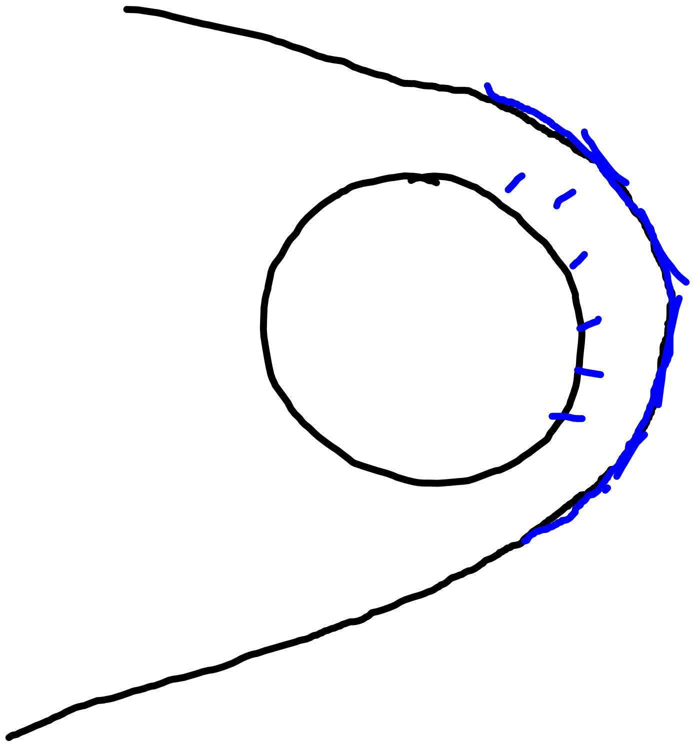
② $\text{grad}(x+2y+3z)$

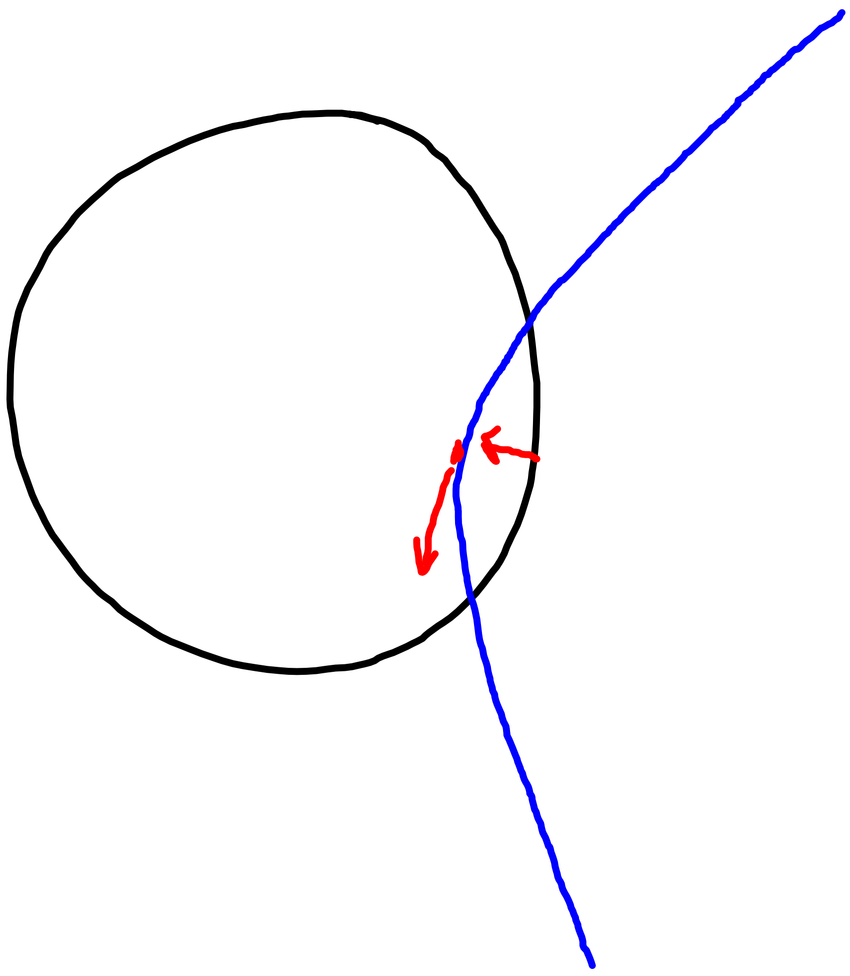
#33





Problematic





§3.2 Flow lines.

scalar field $\phi(x,y,z) = \phi(\vec{R})$

<u>Vector</u> field.

$$\vec{F}(x,y,z) = \vec{F}(\vec{R})$$

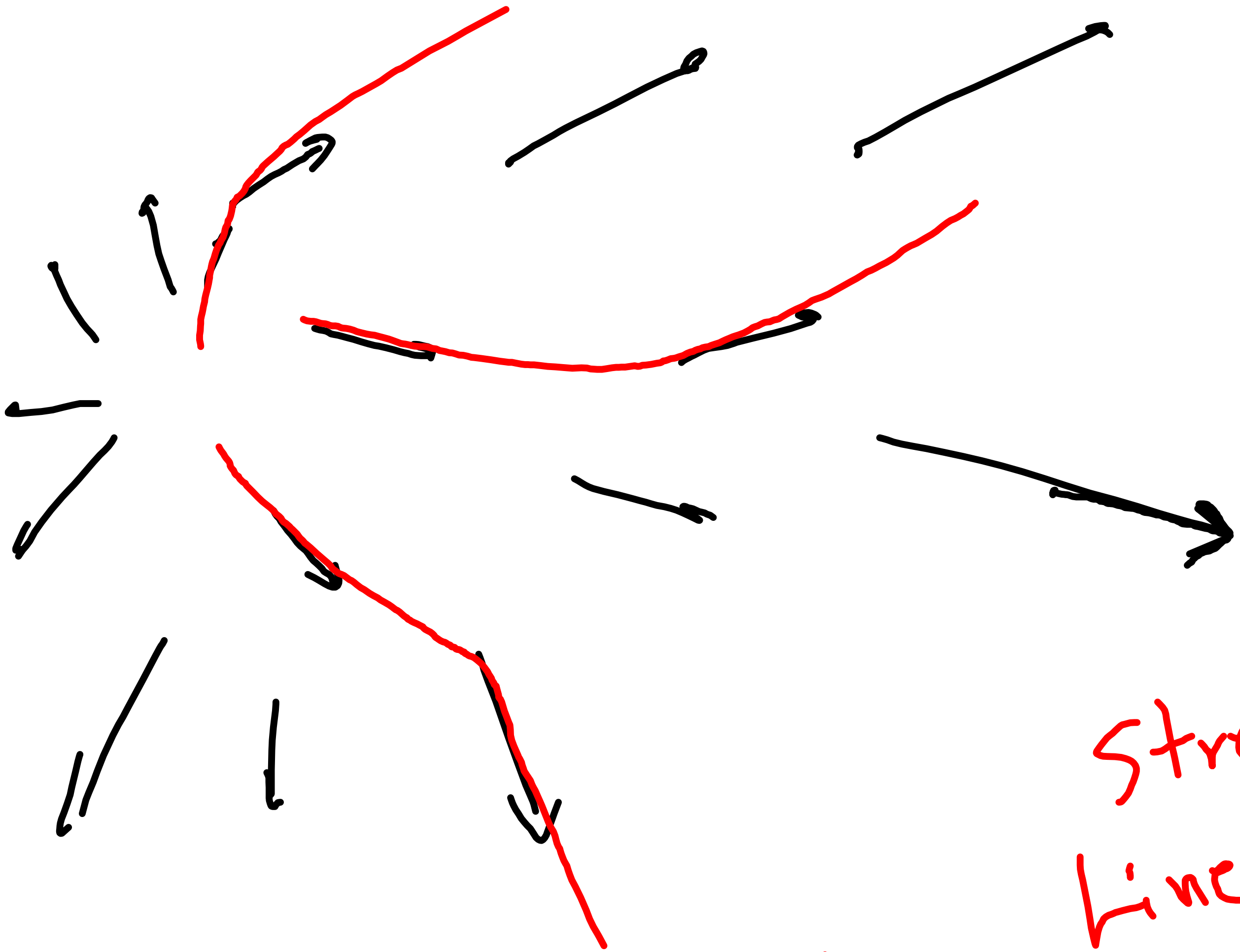
Example. $\operatorname{grad}\phi(\vec{R})$.

electric field $\vec{E}(\vec{R})$. Mag. field $\vec{B}(\vec{R})$.

Flow-velocity field $\vec{V}(\vec{R})$.

Picture.

Vector field.



Streamlines

Lines of flow

A streamline (line of flow) is a curve which is everywhere tangent to the vector field.

$$\text{grad } \phi = \frac{\partial \phi}{\partial x}\vec{i} + \frac{\partial \phi}{\partial y}\vec{j} + \frac{\partial \phi}{\partial z}\vec{k}$$

$$= \vec{\nabla} \phi$$

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\vec{F} = F_1\vec{i} + F_2\vec{j} + F_3\vec{k}$$

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}F_1 + \frac{\partial}{\partial y}F_2 + \frac{\partial}{\partial z}F_3 \text{ : divergence of } \vec{F}$$

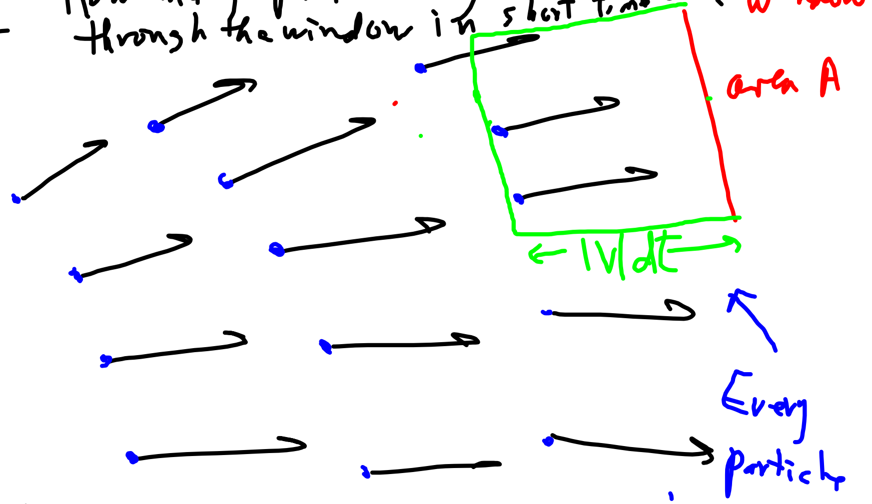
"div $\vec{F}$"

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \overrightarrow{\text{curl } \vec{F}}$$

Flux

How many particles go through the window in short time "dt"?

window



These are velocities

Every particle in this box gets through the window in (dt) sec.

size of box $= (A)(|\vec{v}|\, dt)$

density = # of particles per unit volume = "ν"

So $\nu A |\vec{v}|\, dt$ penetrate in dt

$\nu A |\vec{v}|$ = # of particles per unit time