Gradient:
Scalar field \( \phi \) (voltage, temperature)

\[
\left[ \begin{array}{c}
\text{rate of increase of } \phi \\
\text{w.r.t. distance in a given direction}
\end{array} \right] = \left[ \begin{array}{c}
\text{depends only on the scalar field} \\
\text{depends only on the test direction}
\end{array} \right]
\]

\[
\frac{d\phi}{ds} = (\nabla \phi) \cdot \mathbf{h}_{\text{unit}}
\]

So

1. direction of \( \nabla \phi \) is the direction in which \( \phi \) increases fastest,
2. direction at \( \nabla \phi \) is \( \perp \) (normal) to the level surface of \( \phi \) (isothem, equipotential, ...)
3. \( |\nabla \phi| \) is the highest rate of increase of \( \phi \), over all directions,
\( \phi = \text{voltage} = V(\vec{R}) \)

Sketch

\[ \nabla \phi = \nabla V = -E \]

strongest

\( V = 3 \)

\( V = 2 \)

\( V = 1 \)

\( V = 0 \)
Homework #6

\[ f = \sqrt{x^2 + y^2} \]

Formula for \( f(x, y, z) \):

\[ f = \frac{\frac{x}{r}}{\sqrt{x^2 + y^2}} + \frac{\frac{y}{r}}{\sqrt{x^2 + y^2}} = \frac{x^2 + y^2}{\sqrt{x^2 + y^2}} - 1 \]
#25  
line  \[2x = y = 2z\]

ellipsoid  \[2x^2 + y^2 + 2z^2 = 8\]

What is the angle of intersection?

1. Where does the line intersect the ellipsoid?
2. What is the tangent to the line?
3. What is the normal to the ellipsoid?

Then, use dot product.

Meaning: 90° - (angle between tangent to line and the normal to the ellipsoid)
Normal to ellipsoid: \( 4x\mathbf{i} + 2y\mathbf{j} + \sqrt{2}\mathbf{k} \)