Gradient :

scalar field ϕ (voltage, temperature)

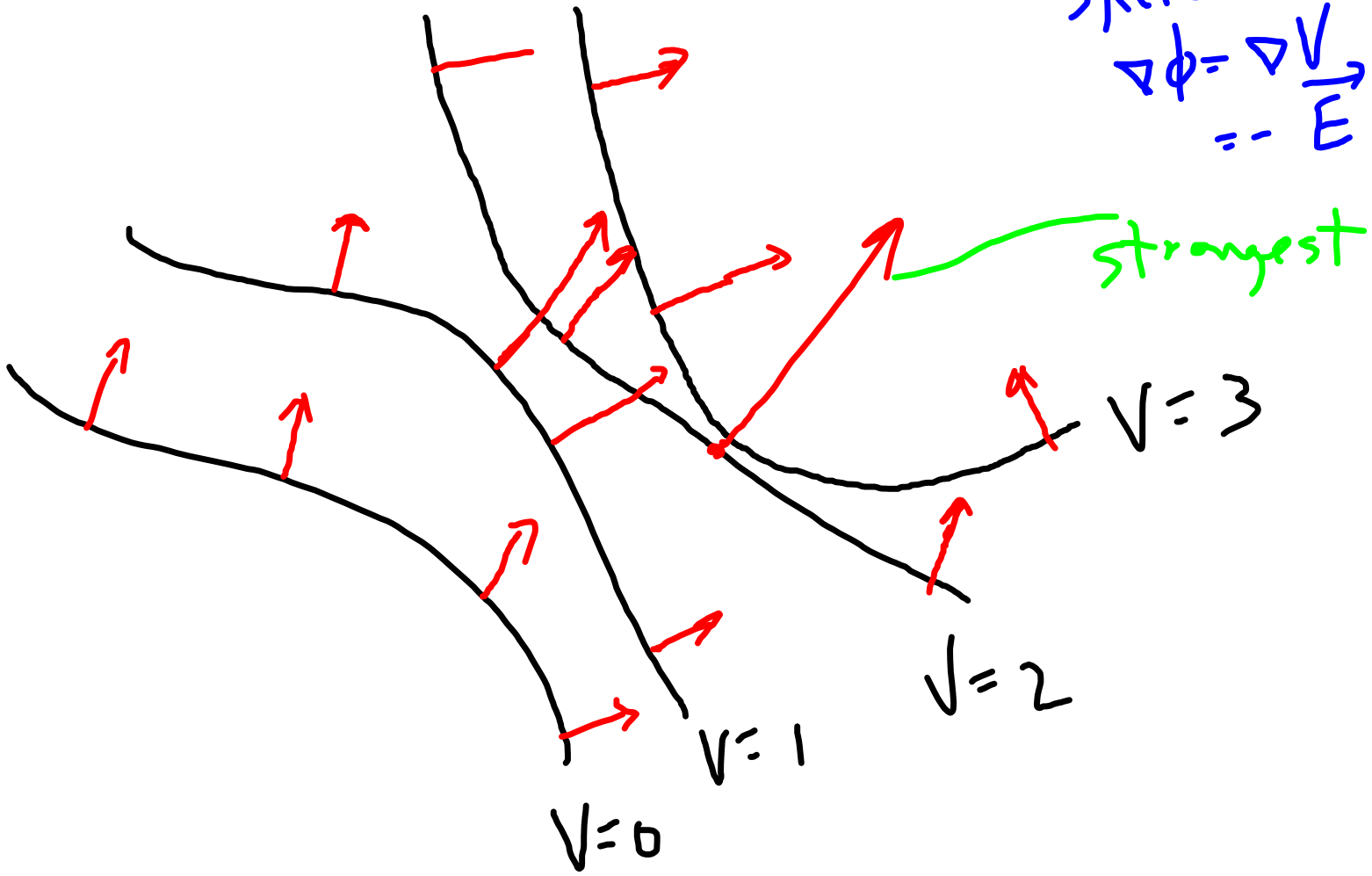
[rate of increase of ϕ w.r.t. distance in a given direction] = [depends only on the scalar field] [depends only on the tgt direction]

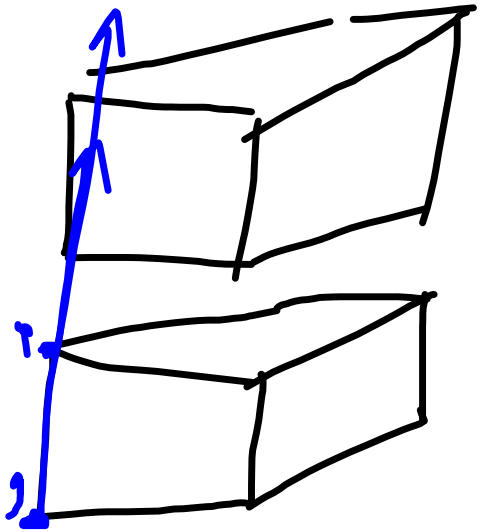
$$\left.\frac{d\phi}{ds}\right|_{\vec{h}} = (\text{grad}\,\phi)\cdot \vec{h}_{\text{unit}}$$

So

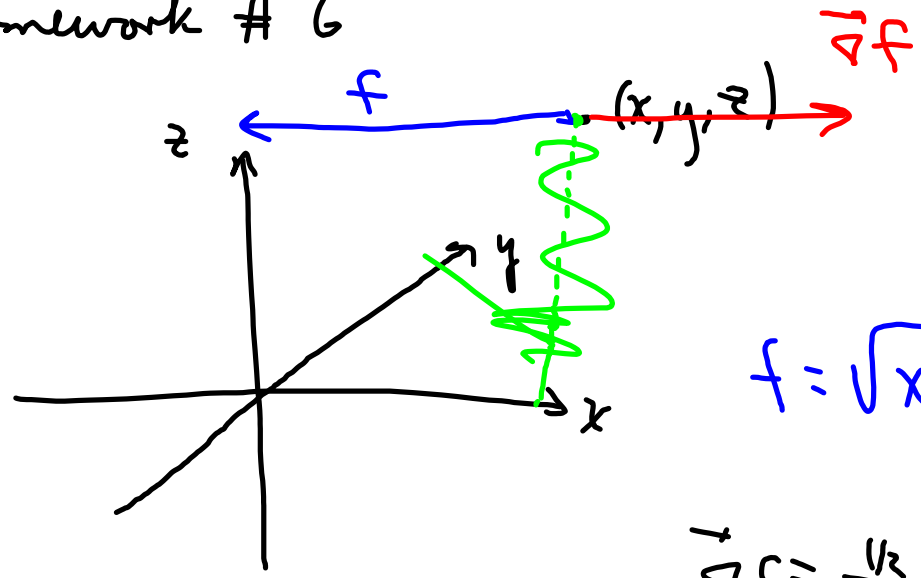
1. direction of $\nabla\phi$ is the direction in which ϕ increases fastest,
2. direction of $\nabla\phi$ is $\perp$ (normal) to the level surface of ϕ (isotherm, equipotential, ...)
3. $|\nabla\phi|$ is the highest rate of increase of ϕ, over all directions,

ϕ = voltage = $V(\vec{R})$





Homework # 6



$\vec{\nabla} f$

$f = \sqrt{x^2+y^2}$

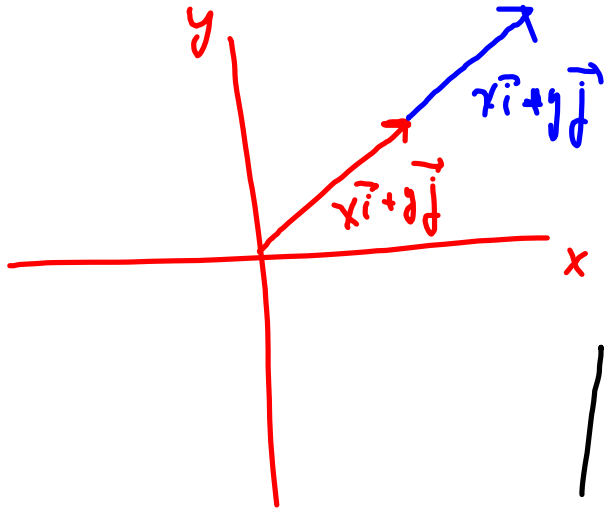
$\vec{\nabla} f = \frac{1/2}{\sqrt{\cdots}} 2x\,\vec{i}$

$+ \frac{1/2}{\sqrt{\cdots}} 2y\,\vec{j}$

$= \frac{x\vec{i}+y\vec{j}}{\sqrt{x^2+y^2}}$

$|\cdots| = 1$

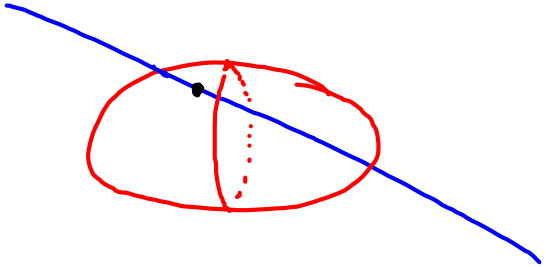
Formula for $f(x,y,z)$



#25 line $2x = y = 2z$

ellipsoid $2x^2 + y^2 + 2z^2 = 8$

What is the angle of intersection?



Meaning:
90° − (angle between tangent to line and the normal to the ellipsoid)

1. Where does the line intersect the ellipsoid?
2. What is the tangent to the line?
3. What is the normal to the ellipsoid?

Then, use dot product.

Normal to ellipsoid $4x\vec{i} + 2y\vec{j} + 4z\vec{k}$