Paratha	\$2	x 2	=	4
Chicken Biriyani	\$8	x 1	=	8
Curried Goat	\$7	x 1	=	7
Firni	\$3	x 2	=	6
				\$25

Dot products arise everywhere in engineering,

In 2D, 3D $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$

Always true: $-1 \le \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|} \le 1$ interpret as a cosine.

$$1+3+4+5+2 \leq \sqrt{1^2+3^2+4^2+5^2+2^2} \cdot$$

$$\sqrt{5}$$

$$\begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

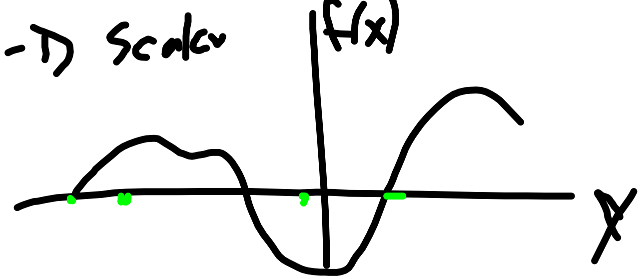
Chpt. 3 Gradient.

Scalar Field. $\phi(x, y, z)$ (Think of temperature)

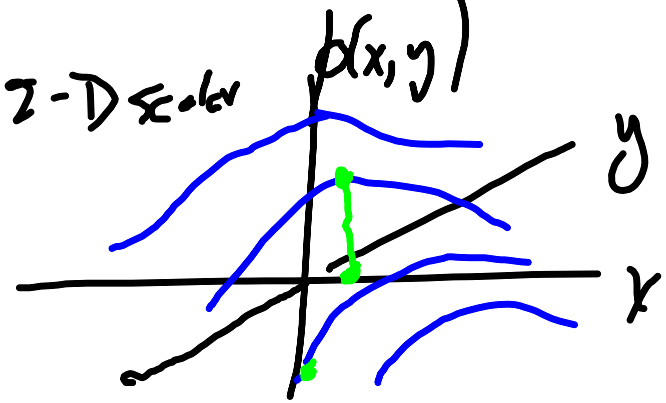
$\phi(\vec{R})$ $\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$

How you plot it?

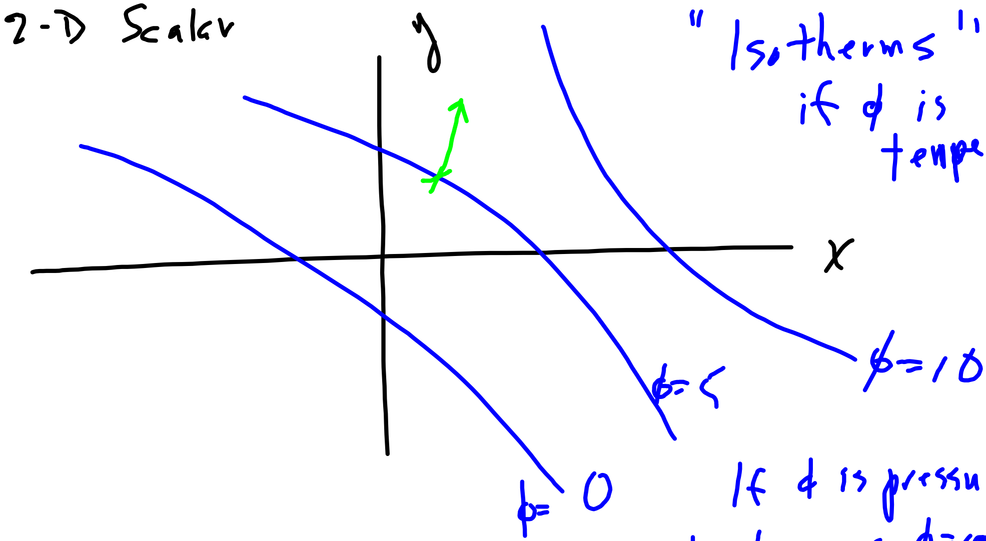
1-D Scalar



2-D Scalar



2-D Scalar



"Isotherms" if ϕ is temperature

If ϕ is pressure level curves ϕ = const are "isobars".

If ϕ is altitude, ϕ = constant are "isoclines"

The generic term is isotonic curves $(\phi(x,y) = \text{const})$

" surfaces $(\phi(x,y,z) = \text{const.})$

Scalar field $\phi(x, y, z)$.

Significance of $\frac{\partial \phi}{\partial x}$? : ~~Rais~~ Rate of increase of ϕ in the x-direction, with respect to distance in the x-direction.

$$\frac{\partial \phi}{\partial y}$$

$$\frac{\partial \phi}{\partial z}$$

What is the rate of increase of $\phi(\vec{R})$, with respect to distance, in an off-axis direction?

Specify the direction? unit vector $\vec{h} = h_1\vec{i} + h_2\vec{j} + h_3\vec{k}$

$$\lim_{s \to 0} \frac{\phi(x+sh_1, y+sh_2, z+sh_3) - \phi(x,y,z)}{s}$$

If $\vec{h} = \vec{i}$, this is $\frac{\partial \phi}{\partial x}$

If $\vec{h} = \vec{j}$, $\frac{\partial \phi}{\partial y}$.

Calculus tells us

$$\phi(x+sh_1,\ y+sh_2,\ z+sh_3)$$

$$\simeq \phi(x,y,z) + \text{"total differential"}$$

$$\frac{\partial\phi}{\partial x}\Delta x + \frac{\partial\phi}{\partial y}\Delta y + \frac{\partial\phi}{\partial z}\Delta z$$

So rate of increase of ϕ in the $\vec{h}$-direction is

$$= \frac{\frac{\partial\phi}{\partial x}sh_1 + \frac{\partial\phi}{\partial y}sh_2 + \frac{\partial\phi}{\partial z}sh_3}{s}$$

"directional derivative"

$$= \frac{\partial\phi}{\partial x}h_1 + \frac{\partial\phi}{\partial y}h_2 + \frac{\partial\phi}{\partial z}h_3$$

$$\vec{h}\cdot\underbrace{\left(\frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}\right)}_{\text{grad}\,\vec{\phi}}$$

Write as $\vec{\nabla}\phi$

$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

"nabla" harp

Δ delta

∇ "del"

Example 3.1.

$$\phi = x^2 + y^2 - z = \phi(x, y, z) = \phi(\vec{R})$$

Find rate of increase of ϕ w.r.t. distance in direction of $4\vec{i} + 4\vec{j} - 2\vec{k}$ at $(1,1,2)$.

Answer: $\overrightarrow{(\text{grad } \phi)} \cdot \vec{n}$

$$\left(2x\vec{i} + 2y\vec{j} - \vec{k}\right) \cdot \frac{4\vec{i} + 4\vec{j} - 2\vec{k}}{\sqrt{16 + 16 + 4}}$$

$x = 1$, $y = 1$

$$= \left(2\vec{i} + 2\vec{j} - \vec{k}\right) \cdot \left(\frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}\right) = \frac{4}{3} + \frac{4}{3} + \frac{1}{3} = \frac{9}{3}$$

$$= 3$$

Scalar field $\phi = xyz$

Point of interest $\vec{R} = 3\vec{i} + 2\vec{j}$

Direction of interest $= \vec{i} + \vec{j} + \vec{k}$

$$\vec{\nabla}\phi = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$$

$$\vec{\nabla}\phi \text{ at } (3,2,0) = 6\vec{k}$$

$$\text{unit direction vector} = \frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{3}}$$

Answer $6/\sqrt{3}$

Break this down.

The gradient was $6\vec{K}$

The rate of increase in direction $\vec{V}$ is

$$6\vec{K} \cdot \frac{\vec{V}}{\|\vec{V}\|} = 6\vec{K} \cdot \vec{h}$$

1. If $\vec{V}$ is horizontal, $\vec{K} \cdot \vec{V} = 0$, no increase, so the isotherm is horizontal here.

2. The optimal direction to go, to get warm fastest, is the direction of the gradient, $6\vec{K}$

3. ______________________, to cool off, ______________________, is $-(6\vec{K})$.

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Summary

1. $\overrightarrow{\text{grad}}\,\phi$ is significant in its own right;

   a. Direction of $\vec{\nabla}\phi$ = the direction of fastest increase of ϕ.

   b. $\vec{\nabla}\phi$ is normal ($\perp$) to the level surface for ϕ (ϕ = const.)

   c. $|\overrightarrow{\text{grad}}\,\phi|$ is the highest rate of increase of ϕ in any direction (and the direction which achieves this is the " of $\vec{\nabla}\phi$.)