Diffusion:

Fick's Law

$$\overrightarrow{\text{flux}} = -k \vec{\nabla} \text{concentration}$$

Cf.

$\overrightarrow{\text{flux}}$ = current density = $\vec{j} = \rho_{\text{charge per electron}} \vec{v}$

$= -e\vec{v}$

$\vec{\nabla} V = (-\vec{E})$

$$\vec{j} = -K \vec{\nabla} V = +\sigma \vec{E}$$

σ — electrical conductivity

For heat

$$\overrightarrow{\text{heat flux}} = -k\,\vec{\nabla}\,T_{\text{emperature}}$$

thermal conductivity,

$$\vec{\nabla}\cdot\overrightarrow{\text{heat flux}} = \text{outflux of heat per unit volume}$$

$$\text{rate at which heat accumulates} = -\vec{\nabla}\cdot\overrightarrow{\text{heat flux}}$$

$$= +k\,\vec{\nabla}^2 T$$

$$\frac{\partial\ \text{energy per unit volume}}{\partial t} = k\nabla^2 T$$

If α ergs flow into a cm^3,

temp increase $= \alpha / C_{\text{spec heat}}$.

$$c\,\frac{\partial T}{\partial t} = k\nabla^2 T \qquad \frac{\partial T}{\partial t} = \frac{k}{c}\nabla^2 T$$

(maybe $k\cdot c$?)

Scotch & Soda Diet.

scotch ~ 150 cal.

soda ~ 0 cal

volume ~ 20 cm^3

weight ~ density × volume ≈ 20 g

initial temperature = 0°

final temperature = 37°

specific heat = 1

heat loss = 20 × 37 = 740 cal.

Vector identities.

curl grad = 0

div curl = 0

$$\vec{\nabla}\cdot(\phi\vec{F}) = \phi\,\nabla\cdot\vec{F} + \vec{F}\cdot\vec{\nabla}\phi$$

$$\vec{\nabla}\times(\phi\vec{F}) = \vec{\nabla}\phi\times\vec{F} + \phi\,\vec{\nabla}\times F$$

Hard ones:

$$\vec{\nabla}\times(\vec{F}\times\vec{G}) = \text{“}\,\vec{\nabla}\cdot\vec{G}\,\vec{F} - \vec{\nabla}\cdot\vec{F}\,\vec{G}\,\text{”}$$

$$= (\vec{\nabla}\cdot\vec{G})\vec{F} - (\vec{\nabla}\cdot\vec{F})\vec{G}$$

$$+ (\vec{G}\cdot\vec{\nabla})\vec{F} - (\vec{F}\cdot\vec{\nabla})\vec{G}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{A}\cdot\vec{C}\,\vec{B} - \vec{A}\cdot\vec{B}\,\vec{C} \checkmark$$

Scratch

$$\vec{F} \cdot \vec{\nabla} \vec{G}$$

$$\vec{F} \cdot \vec{\nabla} G + \vec{F} \cdot \vec{\nabla} \vec{G}$$

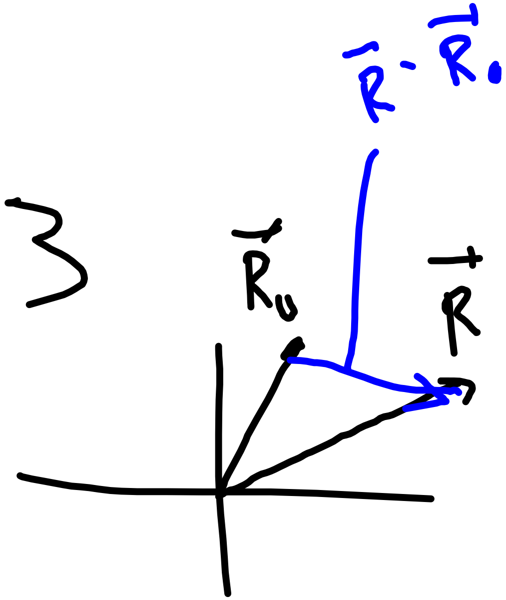
$$(\vec{\nabla} \cdot \vec{F}) \vec{G}$$

$$\vec{\nabla} \cdot \vec{R} = \left(\vec{i}\,\partial_x + \vec{j}\,\partial_y + \vec{k}\,\partial_z\right) \cdot \left(x\vec{i} + y\vec{j} + z\vec{k}\right)$$

$$= 3$$

$$\vec{\nabla} \cdot \vec{R}_0 = 0$$

$$\vec{\nabla} \cdot \left(\vec{R} - \vec{R}_0\right) = 3$$



$$\vec{\nabla} \times \vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ x & y & z \end{vmatrix} = \vec{0}$$

$$\vec{\nabla} \times (\vec{R} - \vec{R}_0) = 0$$

$$\vec{\nabla}(\vec{A}\cdot\vec{R}) = \vec{A}$$

$$\vec{\nabla}\,(A_1\vec{i} + A_2\vec{j} + A_3\vec{k})\cdot(x\vec{i} + y\vec{j} + z\vec{k})$$

$$= \vec{\nabla}(A_1 x + A_2 y + A_3 z)$$

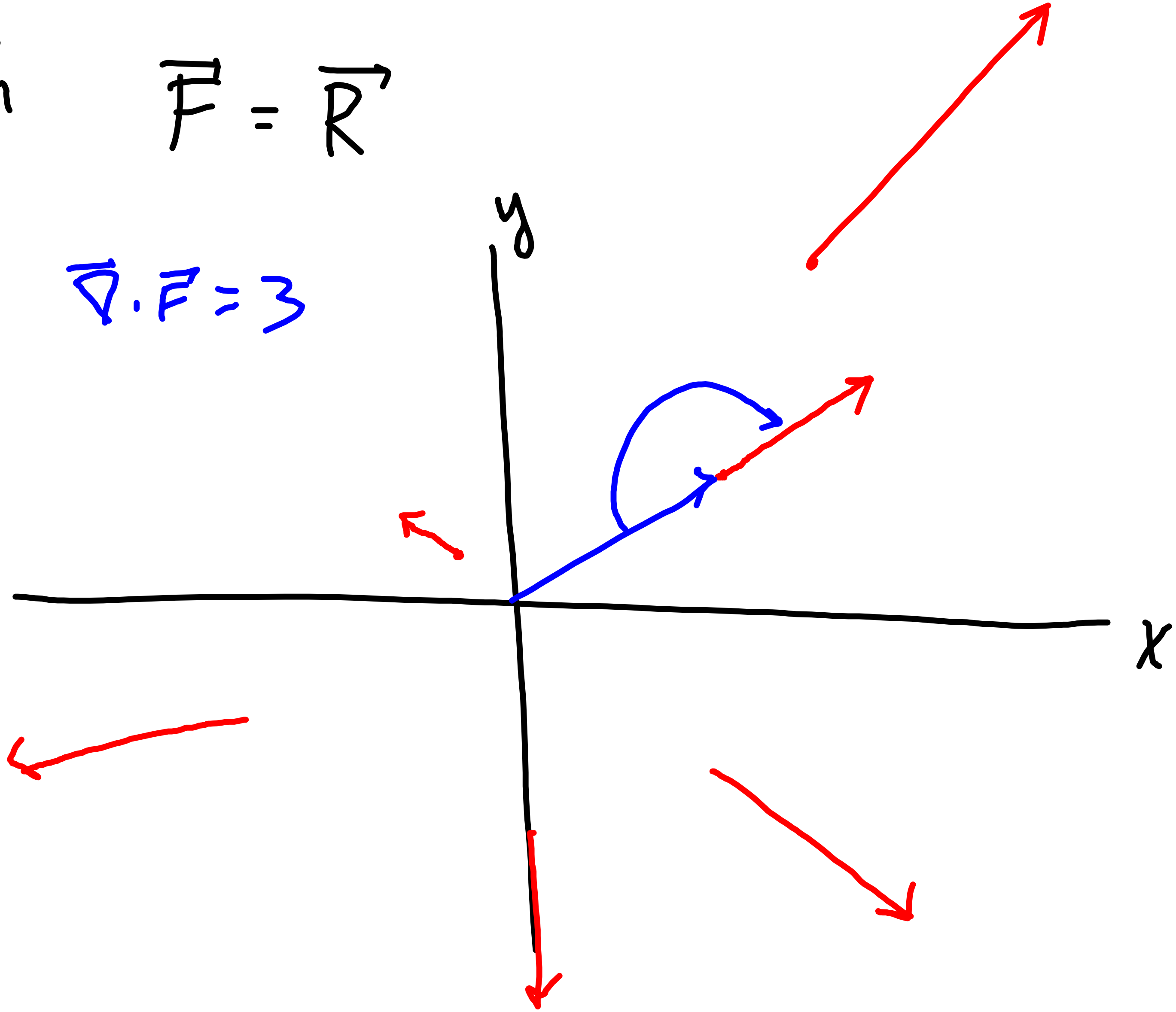
$$= i A_1 + \vec{j} A_2 + \vec{k} A_3$$

$$\vec{F} \cdot (\vec{\nabla} \vec{R})$$ impossible

$$(\vec{F} \cdot \vec{\nabla}) \vec{R}$$

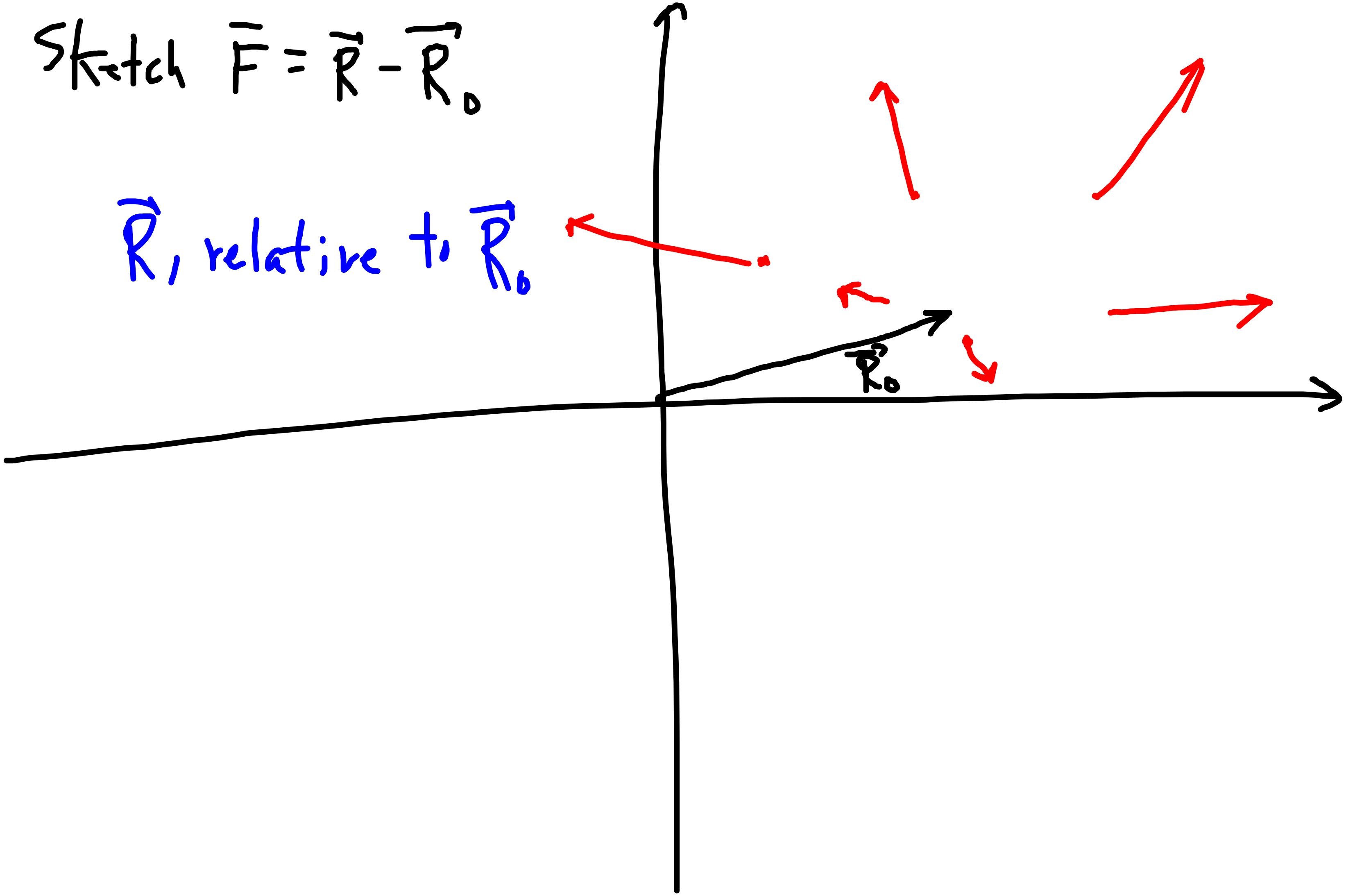
$$\left(F_1 \frac{\partial}{\partial x} + F_2 \frac{\partial}{\partial y} + F_3 \frac{\partial}{\partial z}\right)(x\vec{i} + y\vec{j} + z\vec{k}) = \vec{F} = \vec{F} \cdot \vec{\nabla} \vec{R}$$

Sketch $\vec{F} = \vec{R}$

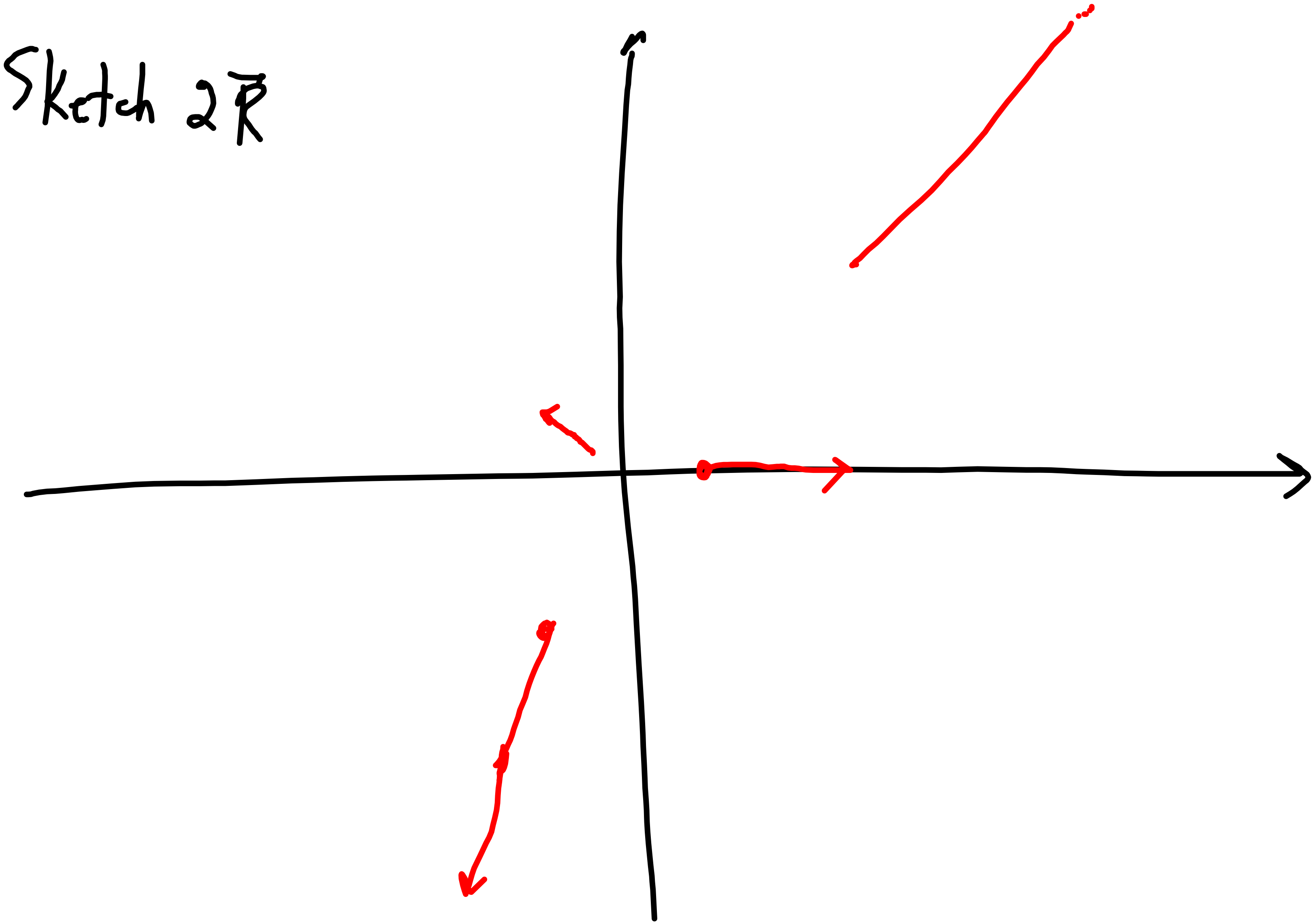


Sketch $\vec{F} = \vec{R} - \vec{R}_0$

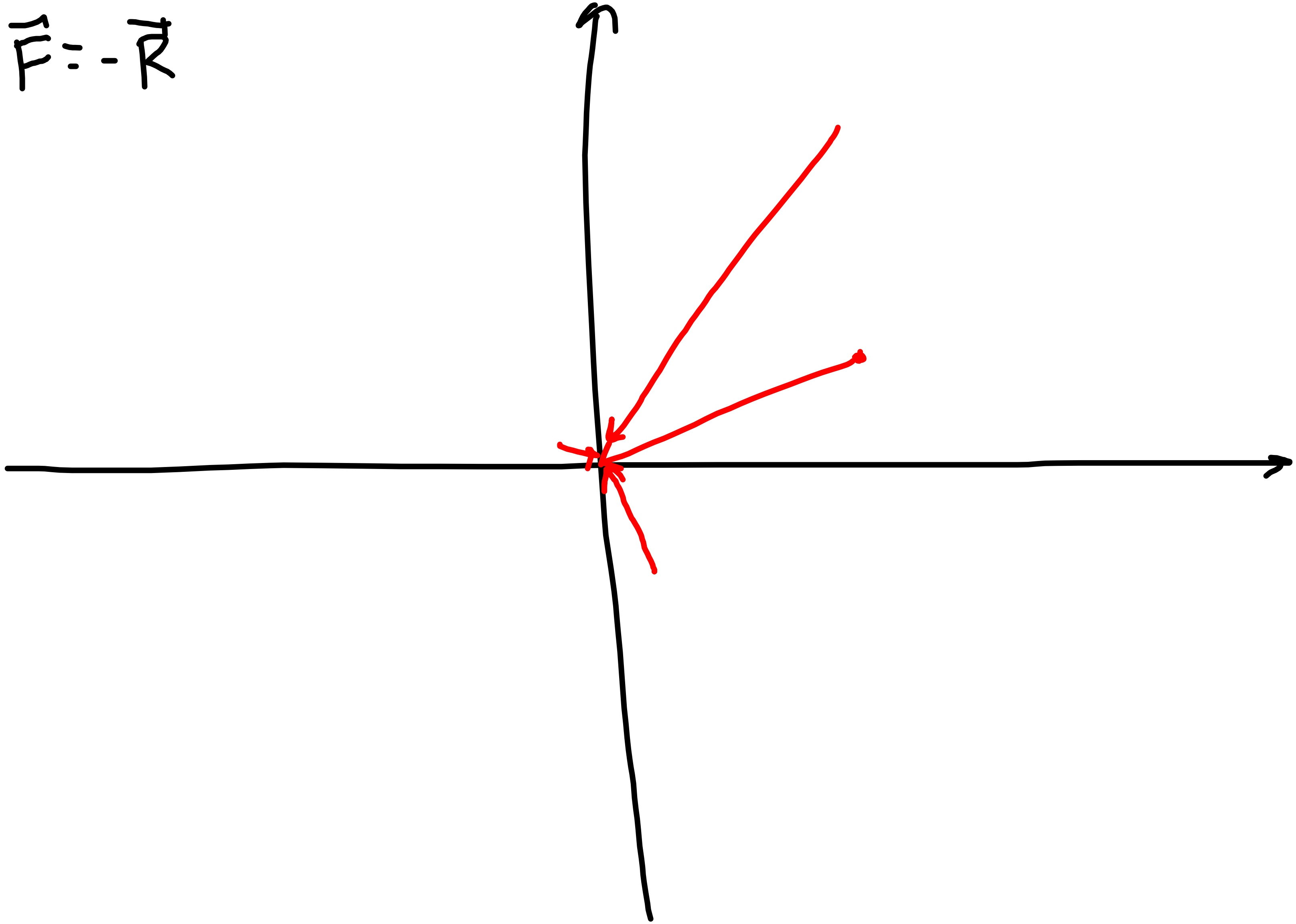
$\vec{R}$, relative to $\vec{R}_0$



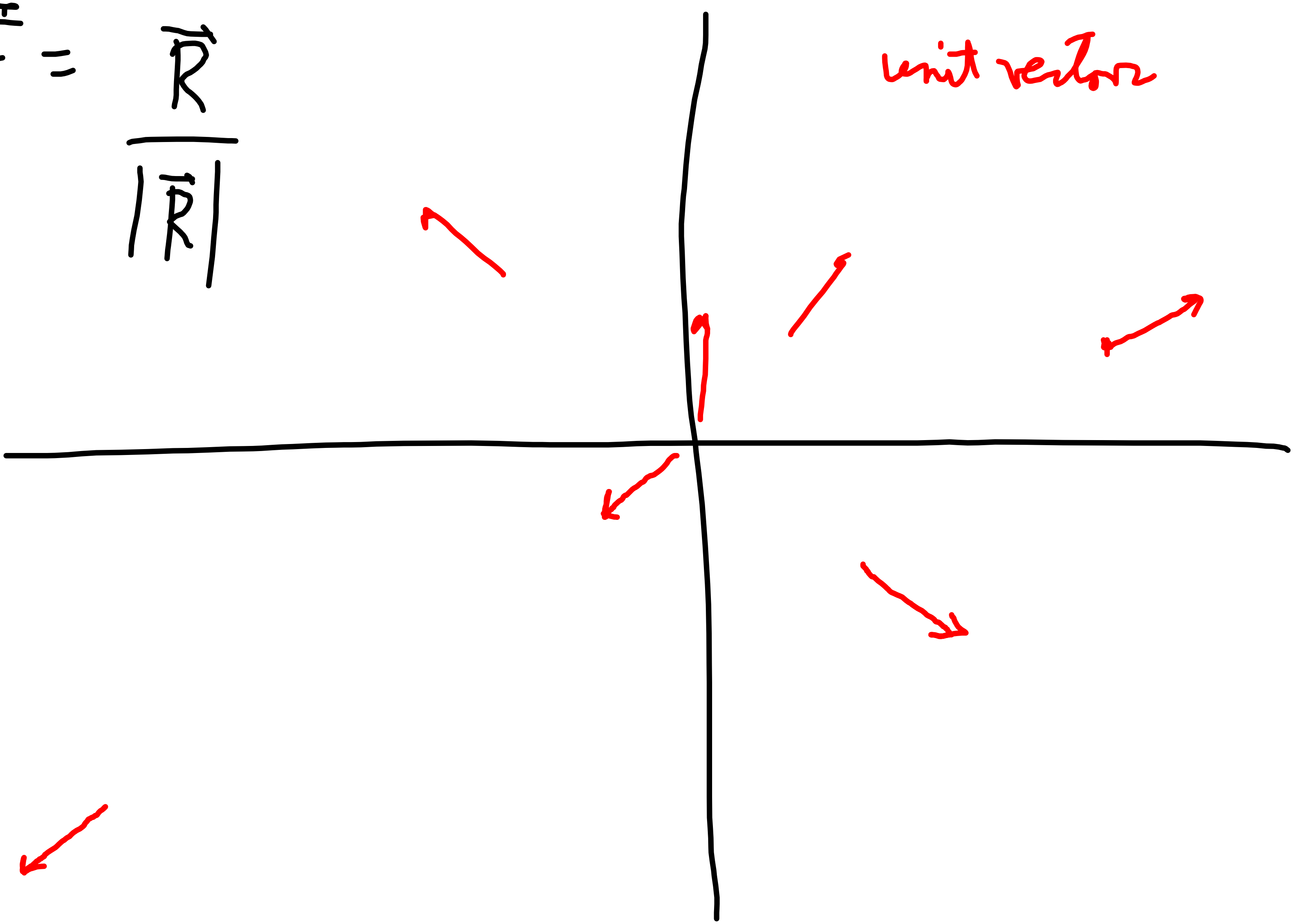
Sketch $2\vec{R}$



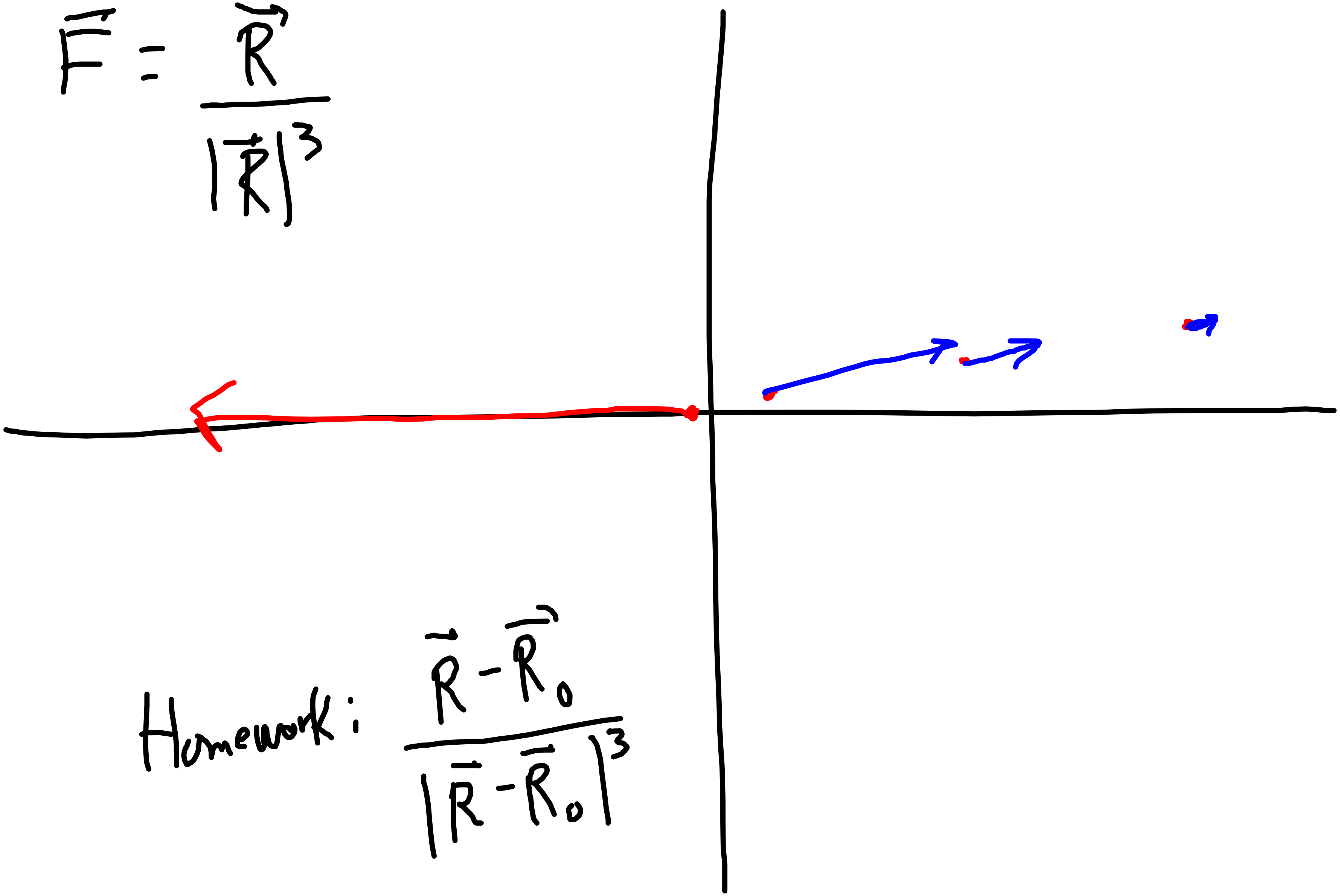
$\vec{F} = -\vec{R}$



$$\vec{F} = \frac{\vec{R}}{|\vec{R}|}$$



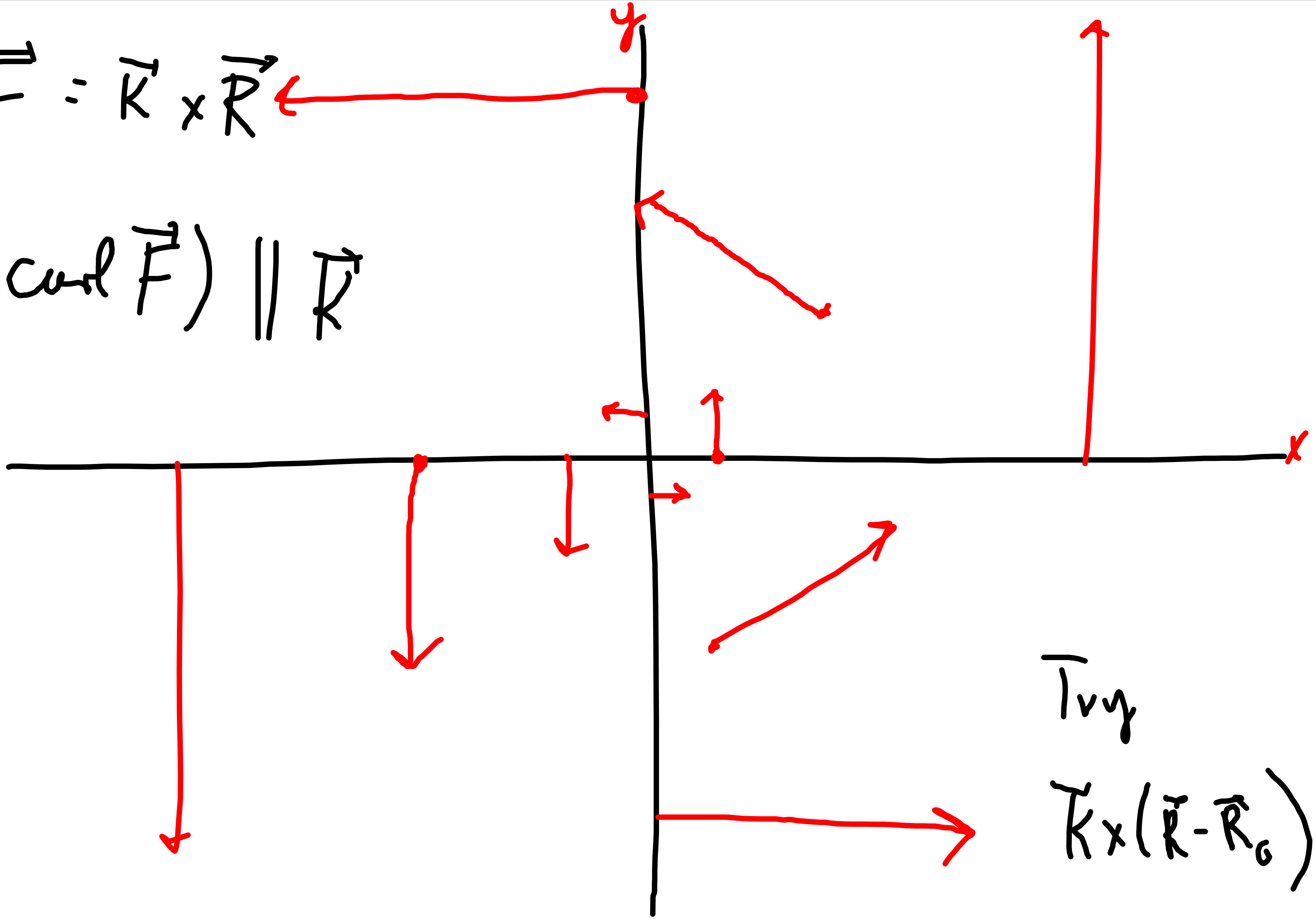
$$\vec{F} = \frac{\vec{R}}{|\vec{R}|^3}$$



Homework: $$\frac{\vec{R} - \vec{R}_0}{|\vec{R} - \vec{R}_0|^3}$$

$\vec{F} = \vec{K} \times \vec{R}$

$(\operatorname{curl} \vec{F}) \parallel \vec{K}$



Try

$\vec{K} \times (\vec{R} - \vec{R}_0)$

Identity

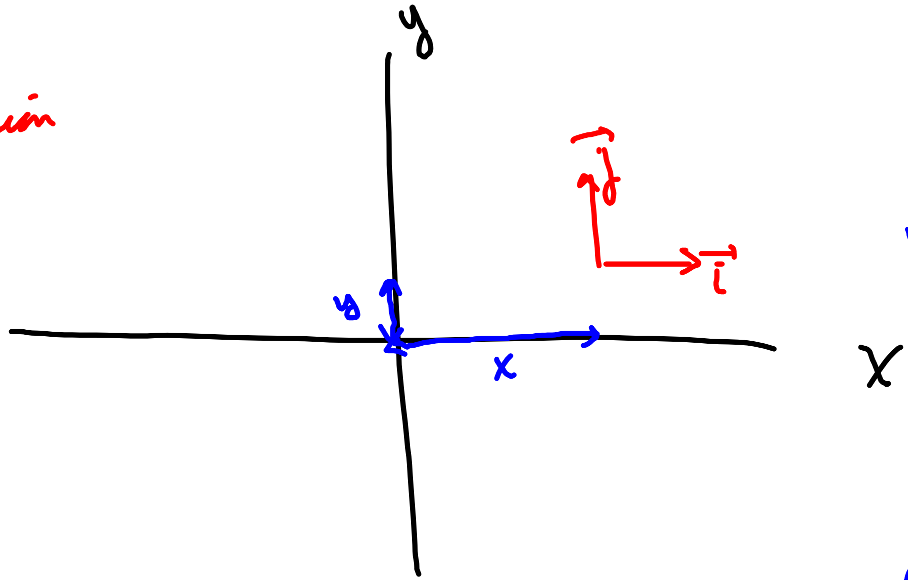
$$\vec{\nabla}(\vec{F}\cdot\vec{G}) = \vec{F}\cdot\vec{\nabla}\vec{G} + \vec{G}\cdot\vec{\nabla}\vec{F}$$

$$+\vec{F}\times(\vec{\nabla}\times\vec{G}) + \vec{G}\times(\vec{\nabla}\times\vec{F})$$

? ? !!

Polar coordinates

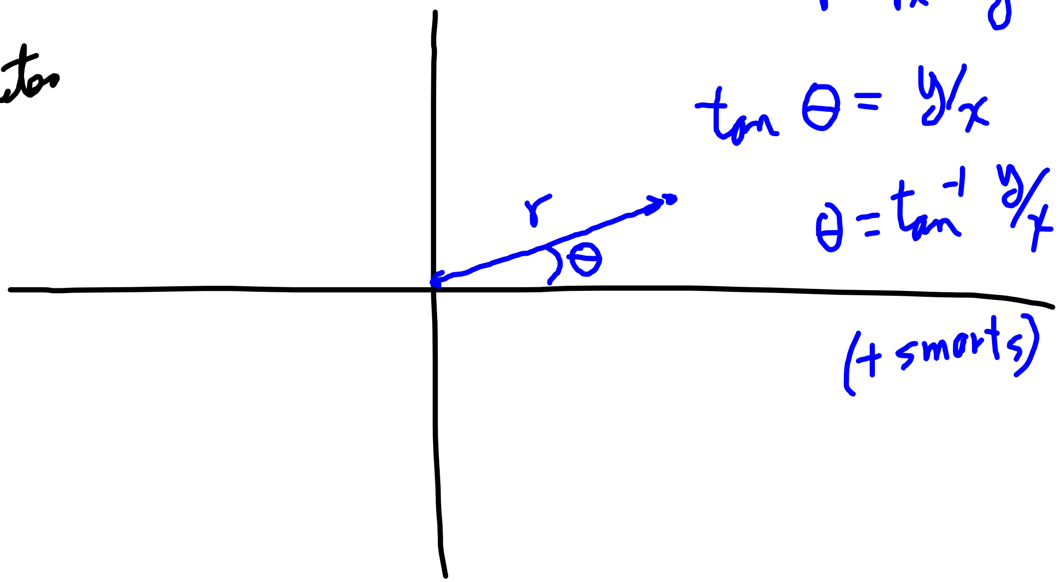
cartesian



$x = r\cos\theta$

$y = r\sin\theta$

polar coordinates

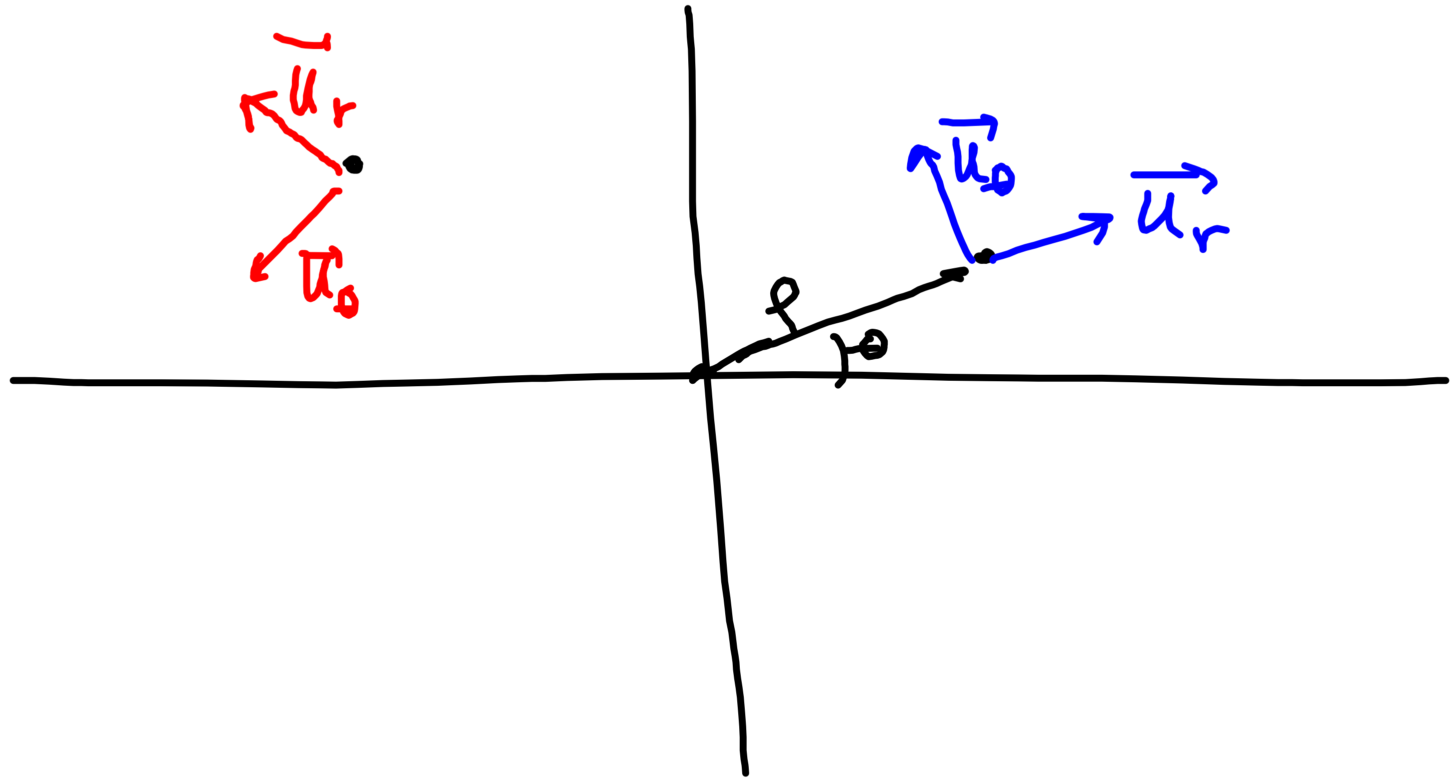


$r = \sqrt{x^2 + y^2}$

$\tan\theta = y/x$

$\theta = \tan^{-1} y/x$

(+ smarts)



$$\frac{\vec{R}}{|\vec{R}|} = \vec{u}_r \qquad \vec{R} = r\,\vec{u}_r \qquad \frac{\vec{R}}{|\vec{R}|^3} = \vec{u}_r / r^2$$

$$\vec{k} \times \vec{R} = r\,\vec{u}_\theta$$