$$\vec{\nabla} = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

∇ scalar $= \nabla\phi$ gradient

$\vec{\nabla}\cdot\vec{F}$ = divergence

$\vec{\nabla}\times\vec{F} = \overrightarrow{\text{curl}}$

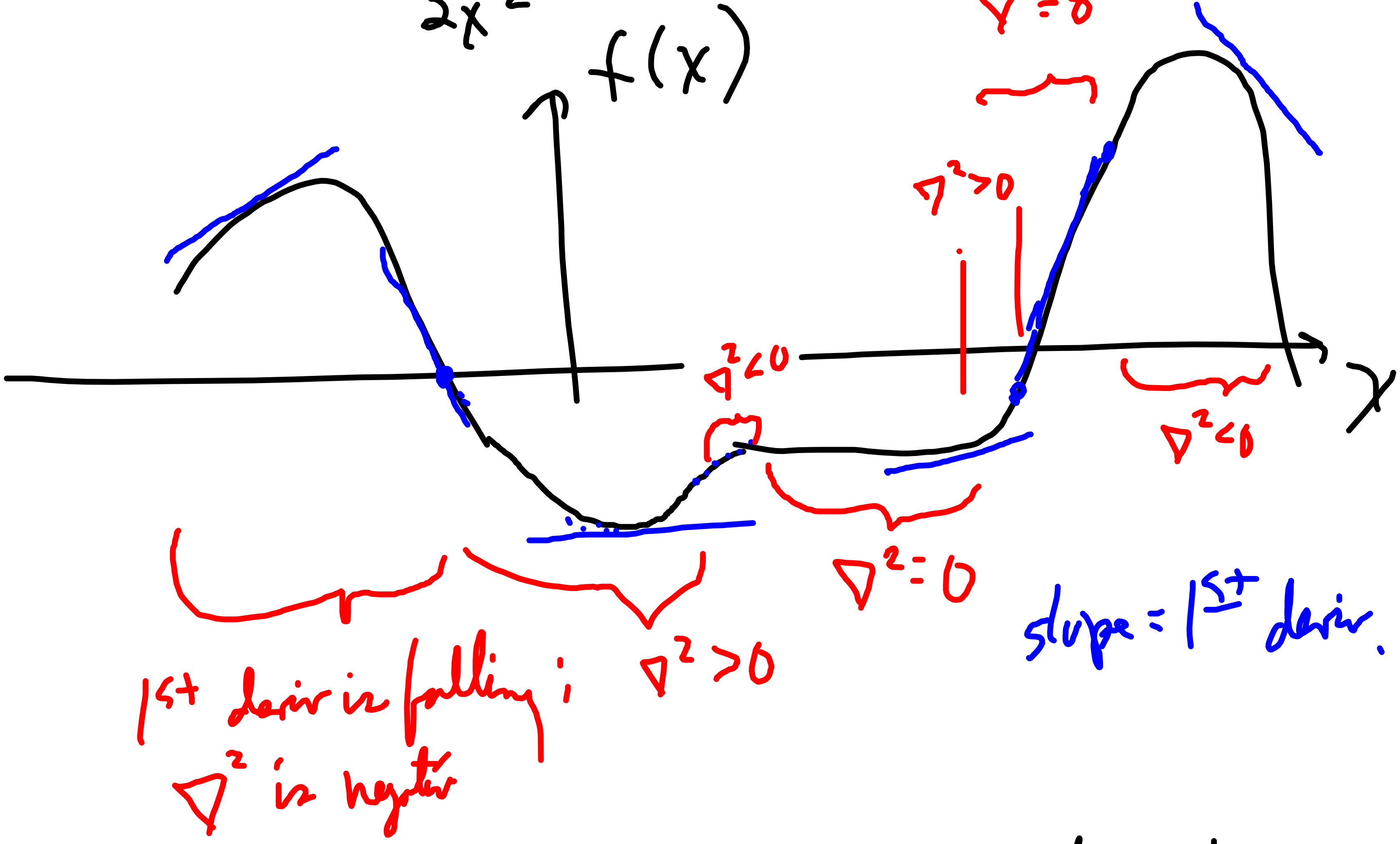
$$\vec{\nabla}\cdot\vec{\nabla}(\text{scalar}) = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$= \vec{\nabla}^2\phi = \Delta\phi$$

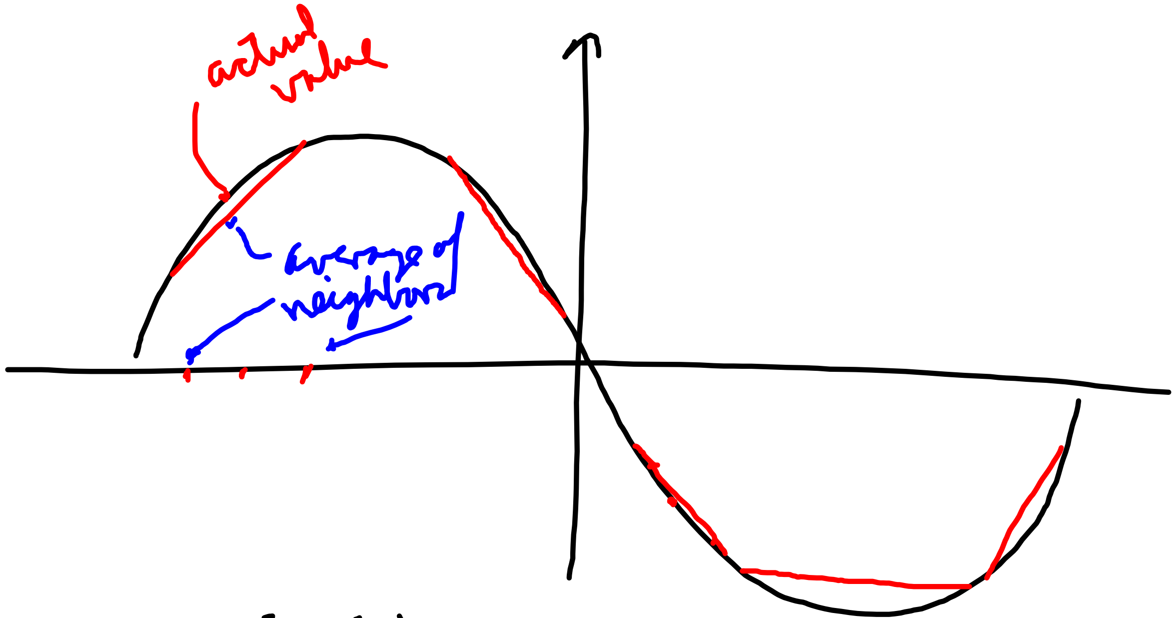
Significance: generalizes the 2^{nd} derivative.

One-dim $\nabla^2 = \frac{\partial^2}{\partial x^2}$ Two dim $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

1-D ∇^2 : $\frac{\partial^2}{\partial x^2} f$



$\nabla^2 = 0$ at inflection points + along straight parts

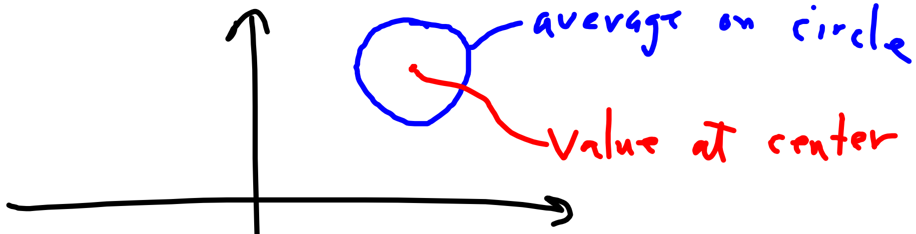


$\nabla^2 f < 0$

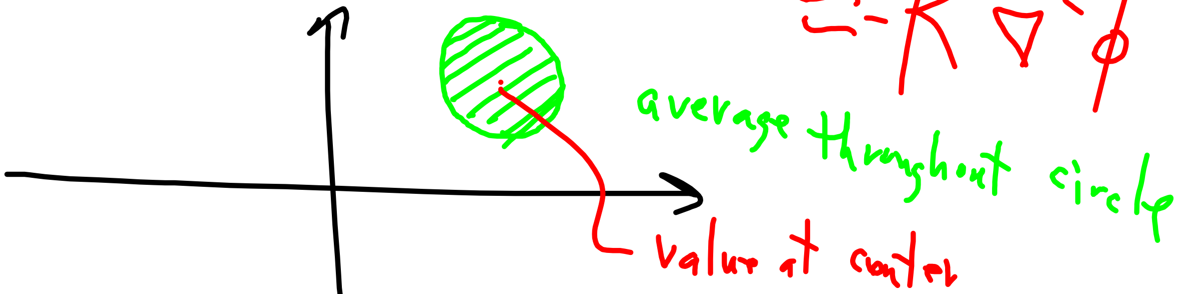
$\nabla^2 f > 0$

Fact: the Laplacian is a rough (negative) measure of the extent to which actual values differ from neighborhood averages.

This is true in 2-D



$$(\text{value at center}) - (\text{average on circle}) = -K \nabla^2 \phi$$



$$(\text{value at center}) - (\text{average throughout circle}) = -K_1 \nabla^2 \phi$$

Heat equation.

$$\frac{\partial T}{\partial t} = C \vec{\nabla}^2 T$$

Hot spots cool off; Cold spots warm up.

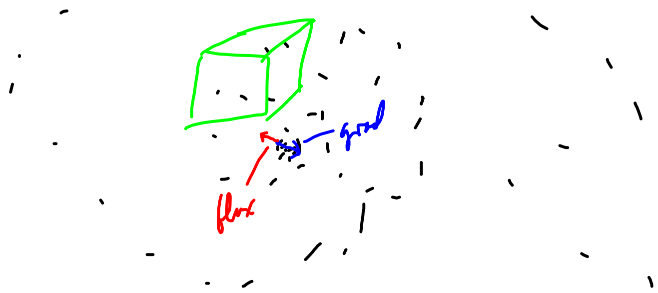
Fick's Law,

Diffusive process.

$\phi(\vec{R}, t)$ is concentration

of whatever's diffusing,

rate of diffusion = flux $= -K \nabla \phi$



rate of increase of population in box

$\propto$ $\underbrace{\text{influx into box}}_{-\vec{\nabla}\cdot(\underbrace{\overrightarrow{\text{flux}}}_{-K\vec{\nabla}\phi})}$

$$\boxed{\frac{\partial \phi}{\partial t} = (+)K \nabla^2 \phi}$$

Diffusion Eq.

(SKip §3.5, §3.7)

§ 3.8 Vector identities.

$$\vec{\nabla} \times \vec{\nabla} \phi = \text{curl grad} \equiv \vec{0}$$

Why?

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \vec{i}\left(\underbrace{\frac{\partial}{\partial y}\frac{\partial \phi}{\partial z}}_{\frac{\partial^2 \phi}{\partial y \partial z}} - \frac{\partial}{\partial z}\frac{\partial \phi}{\partial y} \right)$$

$$\parallel$$

$$0$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = \text{div curl} \equiv 0$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} \cancel{\vec{i}}\,A_1 & \cancel{\vec{j}}\,A_2 & \cancel{\vec{k}}\,A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = \begin{vmatrix} \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix} = \partial_x \partial_y F_3 + \partial_y \partial_z F_1 + \partial_z \partial_x F_2 - \partial_z \partial_y F_1 - \dots$$

$$= 0$$

$$\vec{\nabla}(\phi_1 \phi_2) = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)[\phi_1 \phi_2]$$

$$= \vec{i}\,\phi_1 \frac{\partial \phi_2}{\partial x}$$

$$+ \vec{i}\,\frac{\partial \phi_1}{\partial x}\phi_2 \qquad + \vec{j}_{\,y} + \vec{k} \;\text{''}$$

$$= (\nabla \phi_1)\phi_2 + \phi_1(\nabla \phi_2)$$

$$\vec{\nabla} = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

$$\vec{\nabla}\cdot(\phi\vec{F}) = (\nabla\phi)\cdot\vec{F} + \phi(\nabla\cdot\vec{F})$$

$$\vec{\nabla}\times(\phi\vec{F}) = (\nabla\phi)\times\vec{F} + \phi(\vec{\nabla}\times\vec{F})$$

Bad news

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \vec{\nabla}^2 \vec{F}$$

no physical significance, but straightforward

$$\vec{\nabla}^2 (x^2 \vec{i} + xy \vec{j}) = 2\vec{i}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$