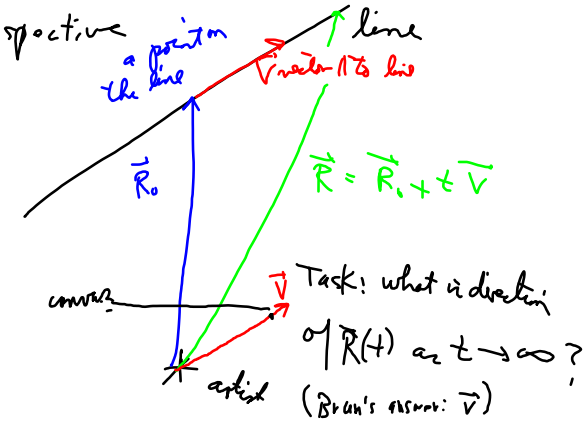
Perspective



Task: what is direction of $\vec{R}(t)$ as $t \to \infty$?

(Brian's answer: $\vec{V}$)

<u>Proof:</u> take a <u>unit</u> vector along $\vec{R}(t)$, and study its limiting value.

$$\frac{\vec{R}(t)}{|\vec{R}(t)|} = \frac{\vec{R}_0 + t\vec{V}}{|\vec{R}_0 + t\vec{V}|} = \frac{\vec{R}_0 + t\vec{V}}{\sqrt{(\vec{R}_0 + t\vec{V})\cdot(\vec{R}_0 + t\vec{V})}}$$

$$= \frac{\vec{R}_0 + t\vec{V}}{\sqrt{\vec{R}_0\cdot\vec{R}_0 + t\vec{V}\cdot\vec{R}_0 + t\vec{R}_0\cdot\vec{V} + t^2\vec{V}\cdot\vec{V}}} \qquad \begin{matrix} 1/t \\ \sqrt{1/t^2} \end{matrix}$$

$$= \frac{\vec{R}_0/t + \vec{V}}{\sqrt{\frac{\vec{R}_0\cdot\vec{R}_0}{t^2} + \frac{1}{t}\vec{V}\cdot\vec{R}_0 \times 2 + \vec{V}\cdot\vec{V}}}$$

as $t \to \infty$

$$= \frac{0 + \vec{V}}{\sqrt{0 + 0 + \vec{V}\cdot\vec{V}}} = \frac{\vec{V}}{|\vec{V}|}$$