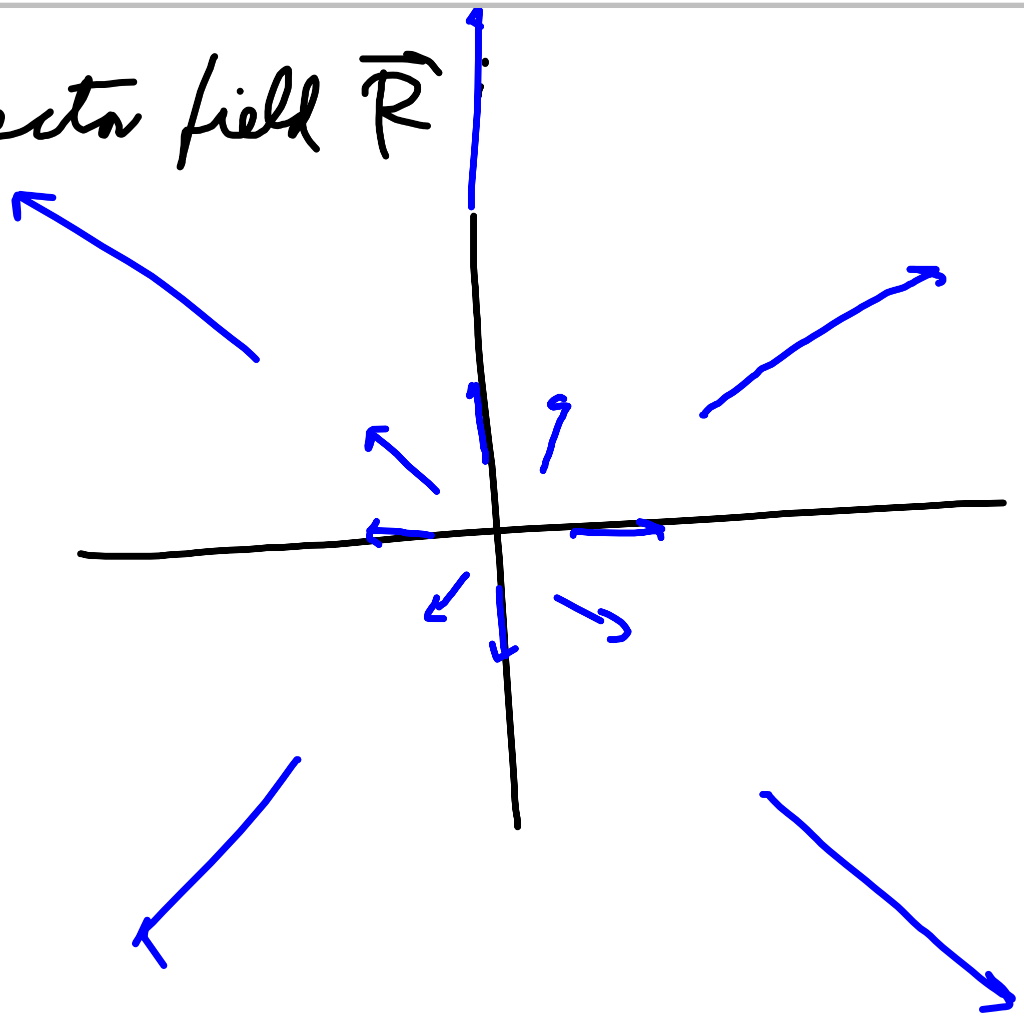
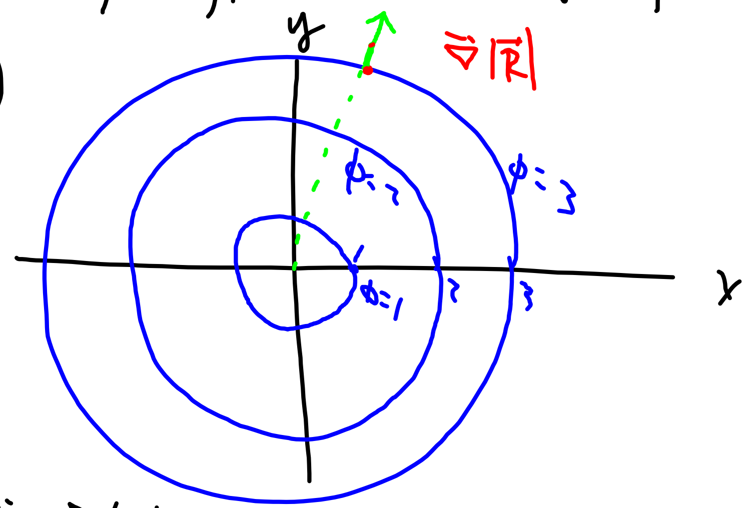
Vector field $\vec{R}$



~~Scla~~ Scalar field

$$\phi(x,y,z) = |\vec{R}| = \sqrt{x^2+y^2+z^2}$$

(2-D)



Direction $\nabla|\vec{R}|$ is obvious.

$$|\nabla|\vec{R}|| = \frac{\partial(\text{scalar})}{\partial(\text{distance along grad})}$$

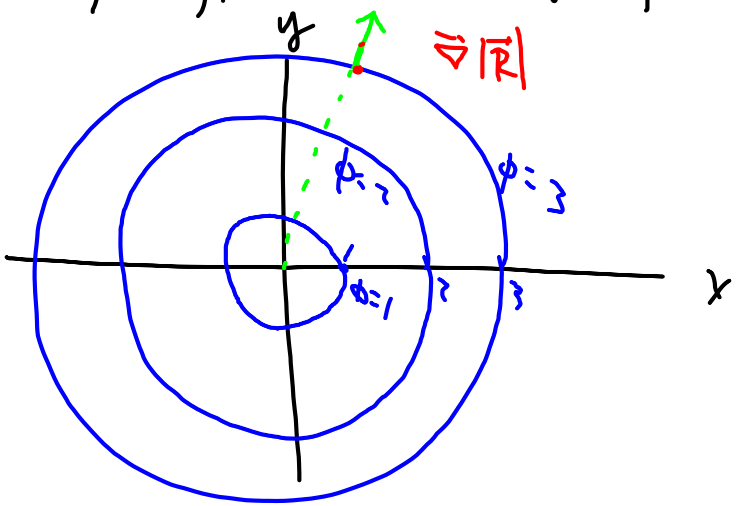
$$= \frac{\partial(\text{distance from origin})}{\partial(\text{distance from origin})} = 1$$

$$\nabla|\vec{R}| = \vec{R}\,\frac{1}{|\vec{R}|} = \frac{\vec{R}}{|\vec{R}|}$$

~~Scal~~ Scalar field

$$\phi(x,y,z) = |\vec{R}| = \sqrt{x^2+y^2+z^2}$$

(2-D)



$\nabla|\vec{R}|^2$ $\left\{ \begin{array}{l} \text{direction} = \vec{R} \\ |\nabla|\vec{R}|^2| = \dfrac{2(\text{distance}^2)}{\partial\ \text{distance}} \end{array} \right.$

$= 2(\text{distance})$

$= 2|\vec{R}|$

$$\nabla |\vec{R}|^n = \frac{\vec{R}}{|\vec{R}|} \, n \, |\vec{R}|^{n-1}$$

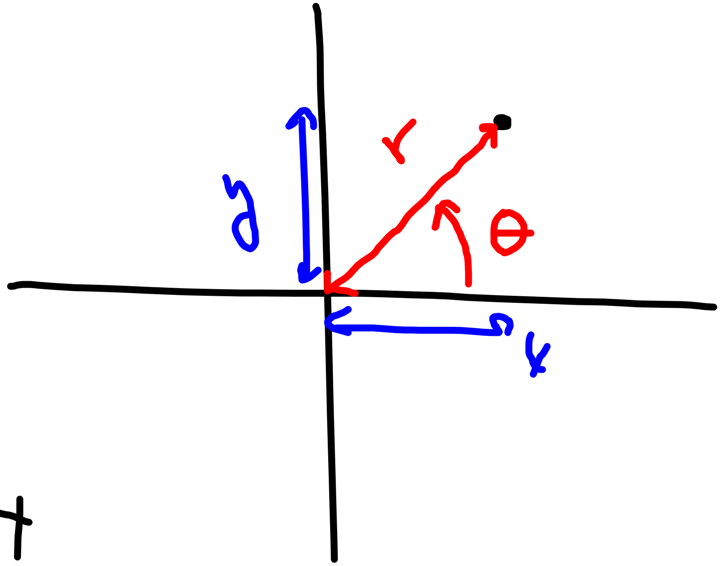
$$\vec{\nabla} |\vec{R}|^n = n \vec{R} \, |\vec{R}|^{n-2}$$

works for $\pm n$.

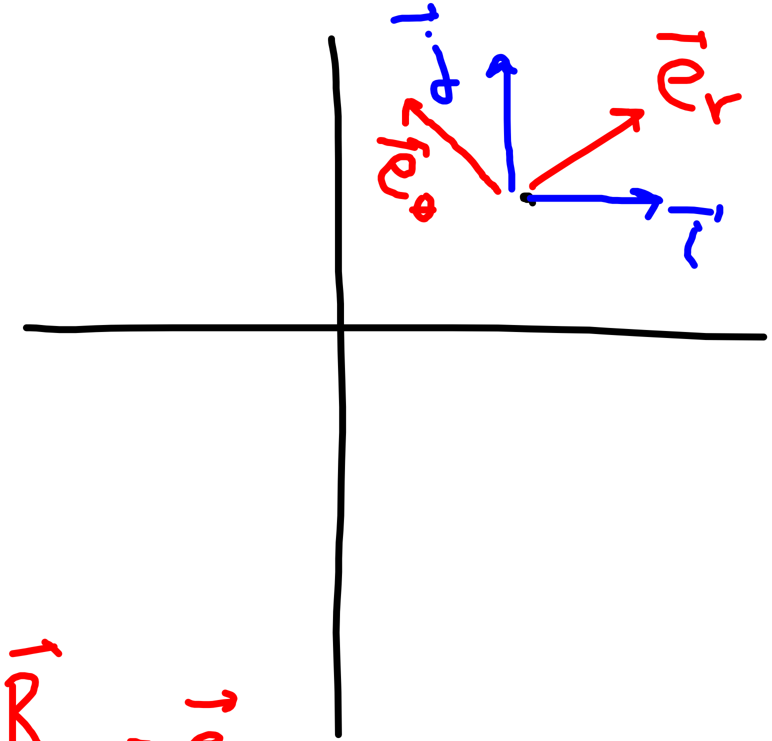
$$\vec{\nabla} \frac{1}{|\vec{R}|^n} \stackrel{?}{=} (-n) \vec{R} \, |\vec{R}|^{-n-2} = \vec{\nabla} |\vec{R}|^{-n}$$

$$\vec{\nabla} |\vec{R} - \vec{R}_0|^n = n (\vec{R} - \vec{R}_0) \, |\vec{R} - \vec{R}_0|^{n-2}$$

Polar coordinates

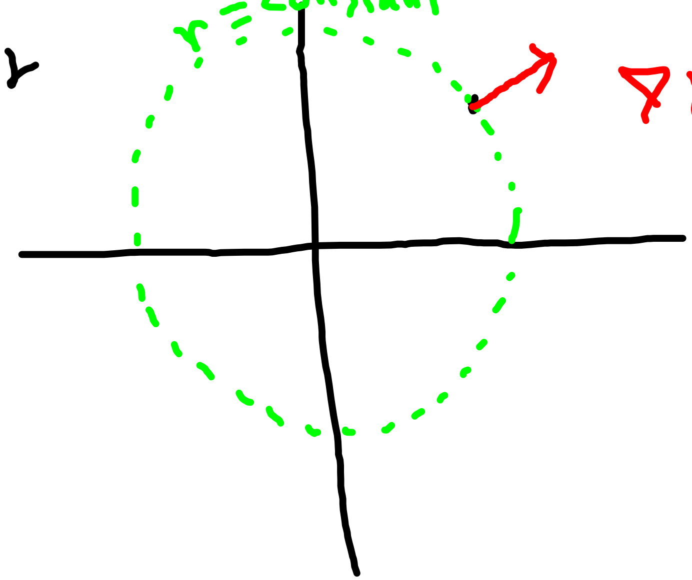


Unit vectors



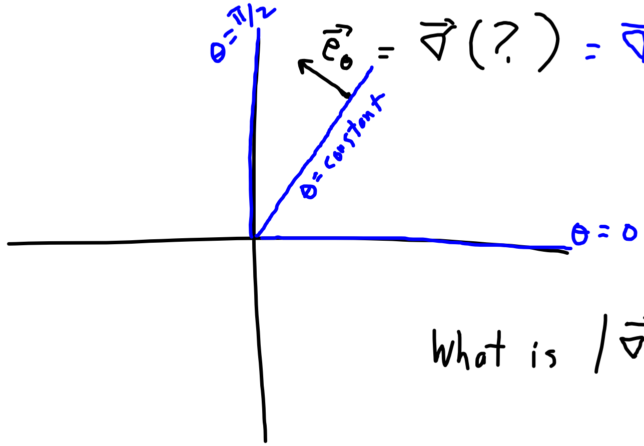
Look at scalar

$|\vec R| = r$



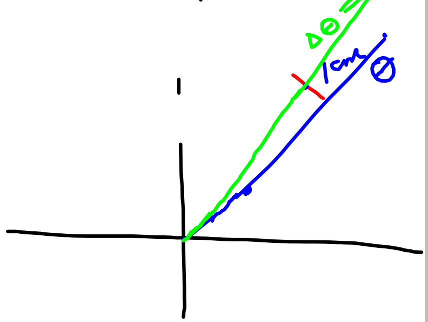
$$\nabla r = \frac{\vec R}{|\vec R|} = \vec e_r$$

$\vec{e}_\theta = \vec{\nabla}(?) = \vec{\nabla}\theta$ in direction



level curves of θ

What is $|\vec{\nabla}\theta|$?



radians: $r\theta$ = distance

$$\theta = \frac{\text{distance}}{r}$$

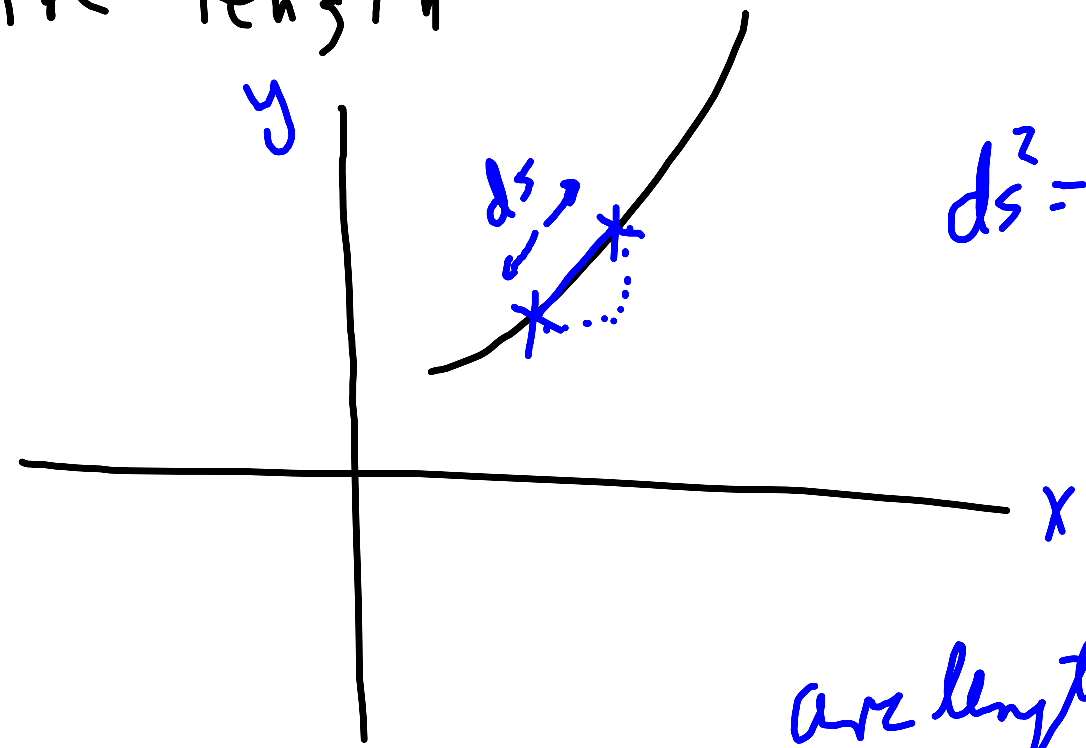
$$|\vec{\nabla}\theta| = \frac{\text{increase in }\theta}{\text{distance}} = \frac{1}{r}$$

$\vec{e}_\theta$ = unit vector in $\vec{\nabla}\theta = \dfrac{\vec{\nabla}\theta}{1/r} = r\vec{\nabla}\theta$

Alternative interpretations:

$$\vec{e}_r = \nabla r \qquad \vec{e}_\theta = (\vec{\nabla}\theta)\, r$$

Arc length



$$ds^2 = dx^2 + dy^2$$

$$\text{arc length} = \int_{\text{start}}^{\text{end}} \sqrt{dx^2 + dy^2}$$

For a trajectory,

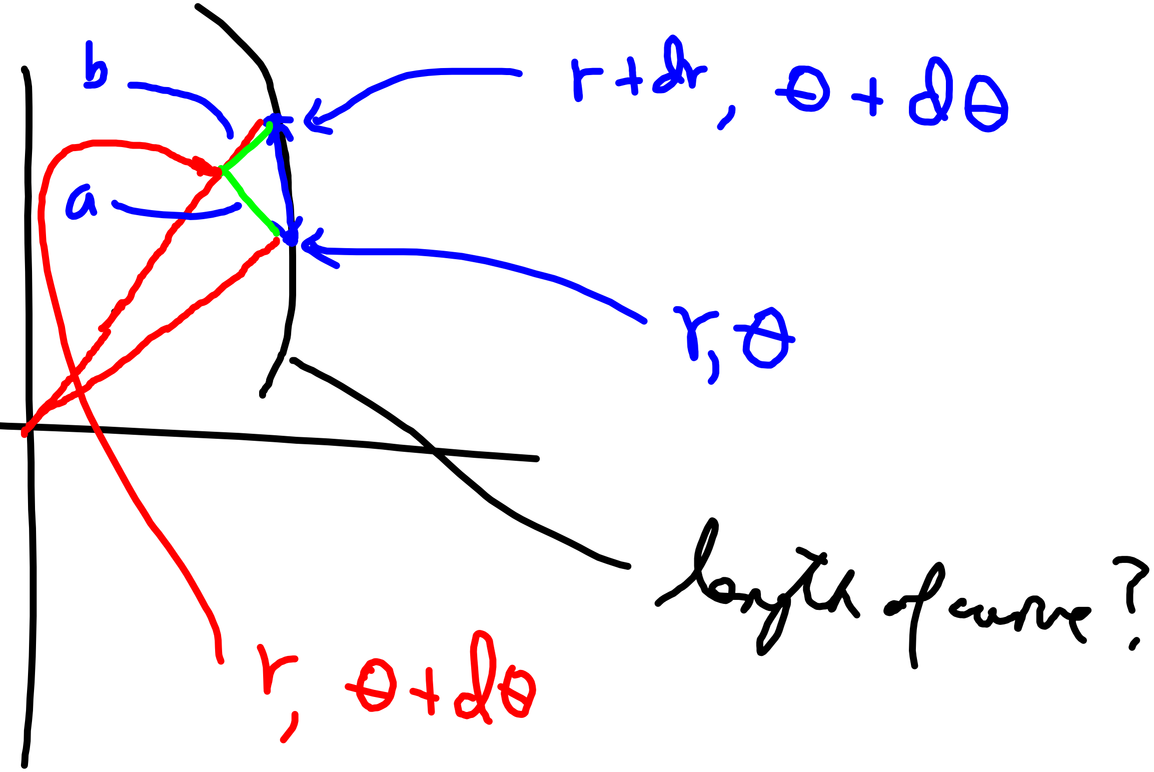
$x = x(t)$

$y = y(t)$

$$\text{length} = \int_{\text{start}}^{\text{end}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$ds^2 = a^2 + b^2$$

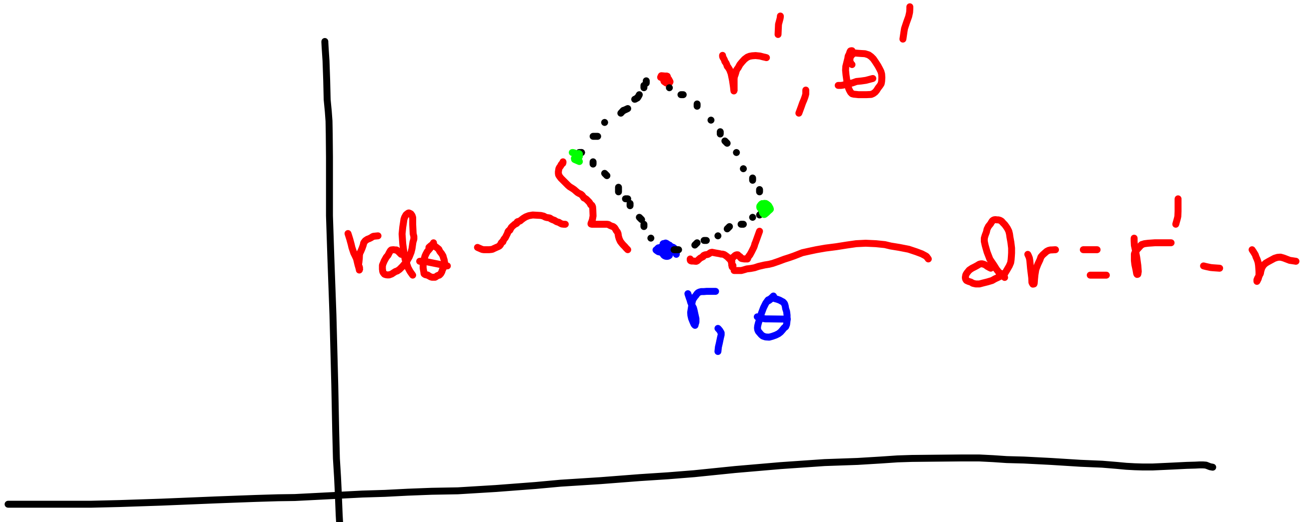
$$b = dr$$



$$a = r\,d\theta$$

$$\therefore\ ds^2 = (dr)^2 + r^2(d\theta)^2$$

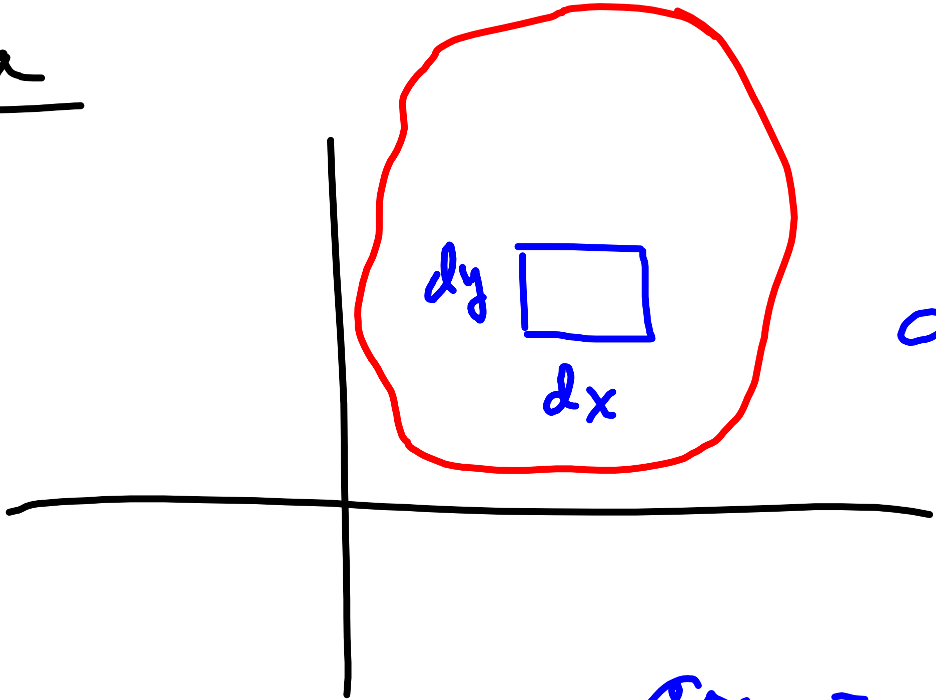
$$\text{length} = \int_{\text{start}}^{\text{finish}} \sqrt{(dr)^2 + r^2(d\theta)^2}$$



distance between (r,θ) & (r',θ')

$$= \sqrt{(dr)^2 + r^2(d\theta)^2}$$

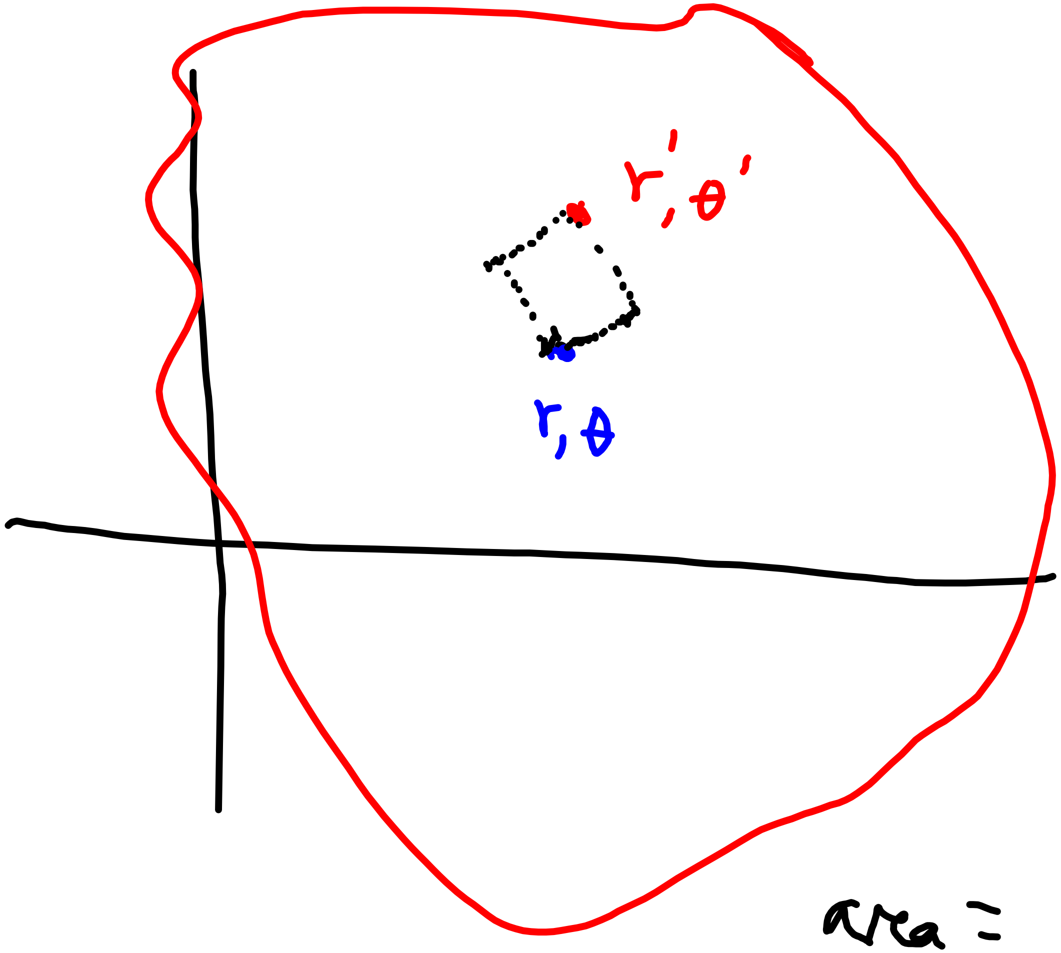
Area



$$dA = dx\,dy$$

$$\text{area} = \iint dx\,dy$$

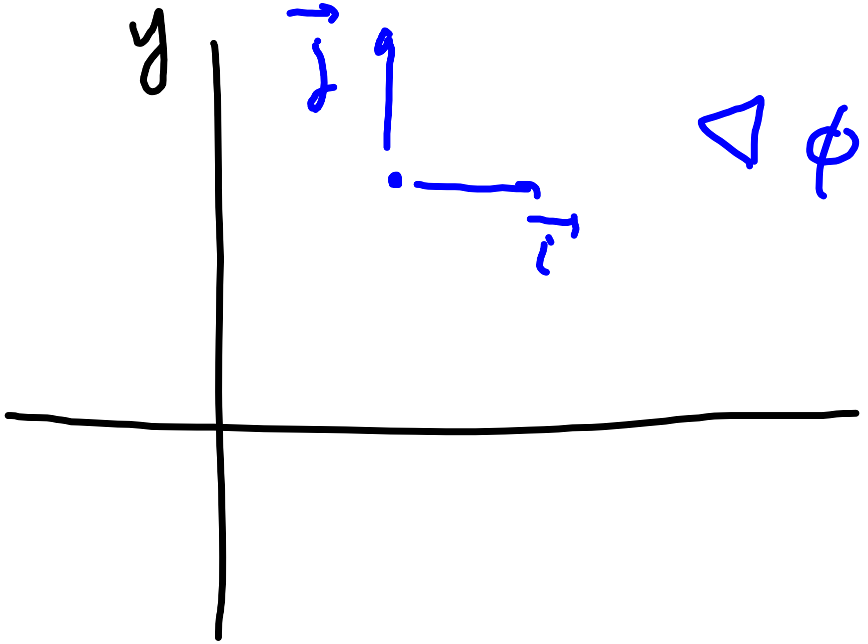
Area in polar coordinates



$$dA = (dr)(r\,d\theta)$$

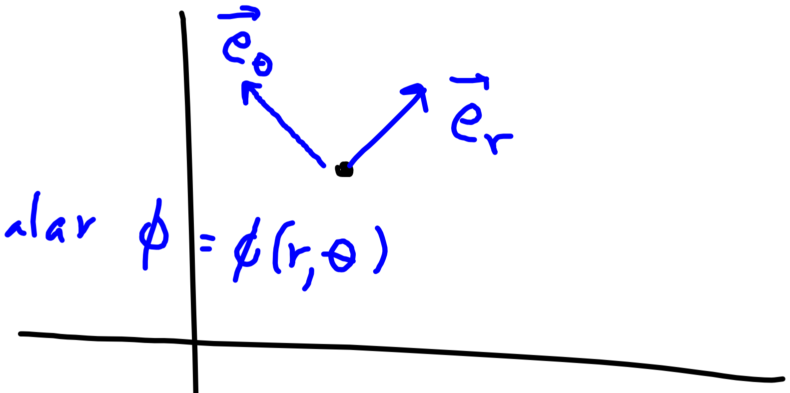
$$\text{area} = \iint r\,dr\,d\theta$$

gradient



$$\nabla \phi = \frac{\partial \phi}{\partial x}\vec{i} + \frac{\partial \phi}{\partial y}\vec{j}$$

gradient in polar coordinates



Scalar $\phi = \phi(r,\theta)$

$$\vec{\nabla}\phi = \frac{\partial \phi}{\partial(\text{distance along } \vec{e}_r)}\vec{e}_r + \frac{\partial \phi}{\partial(\text{distance along } \vec{e}_\theta)}\vec{e}_\theta$$

$$= \frac{\partial \phi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\vec{e}_\theta$$

Cylindrical coords.

$$x, y, z \longleftrightarrow \cancel{r}\ \rho, \theta, z$$

$$\phi = \frac{z}{\rho^2} = \phi(\rho, \theta, z)$$

$$\vec{\nabla}\phi = \frac{\partial\phi}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial\phi}{\partial\theta}\vec{e}_\theta + \frac{\partial\phi}{\partial z}\vec{e}_z$$

$$= -2\frac{z}{\rho^3}\vec{e}_\rho + (0) + \frac{1}{\rho^2}\vec{e}_z$$