Vector properties.

If $\nabla \times \vec{F} = \vec{0}$, can replace $\vec{F}$ by $\vec{\nabla}\phi$.

If $\nabla \cdot \vec{F} = 0$, can replace $\vec{F}$ by $\vec{\nabla} \times \vec{A}$ + can impose one condition on $\vec{A}$.

(Because you add any grad $\vec{\nabla}\psi$ to $\vec{A}$ + $\vec{F}' = \nabla \times \vec{A}'$.)

"Gauge" transformation.

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \vec{D} = \epsilon \vec{E}$$

ρ charge density

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

$\vec{j}$ = current flux $= \rho \vec{v}$

$$\vec{H} = \vec{B}/\mu$$

Conservation charge

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{j} = i\omega\rho$$

Replace $\vec{D}$, $\vec{H}$ by $\epsilon\vec{E}$, $\vec{B}/\mu$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon$$

In frequency domain: all $\propto e^{i\omega t}$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad = -i\omega\vec{B}$$

$$\vec{\nabla}\times\vec{B} = \mu\vec{j} + \mu\epsilon\frac{\partial\vec{E}}{\partial t} \qquad = \mu\vec{j} + i\omega\mu\epsilon\vec{E}$$

Redundancy in M's eqs.

take div of $\quad \vec{\nabla} \times \vec{H} = \vec{j} + i\omega \vec{D}$

$$\nabla \cdot \text{curl} = 0 = \nabla \cdot \vec{j} + i\omega \vec{\nabla} \cdot \vec{D}$$

$$\nabla \cdot \vec{j} = -i\omega\rho$$

$i\omega\rho = i\omega \vec{\nabla} \cdot \vec{D}$, so $\vec{\nabla} \cdot \vec{D} = \rho$ follows, if $\omega \neq 0$.

So ~~the~~ 1st eq is redundant for AC, but not for statics DC.

2nd eq.

$\vec{\nabla} \cdot \vec{B} = 0$ so $\vec{B}$ can be replaced by $\vec{\nabla} \times \vec{A}$.

3rd eq.

$$\vec{\nabla} \times \vec{E} = -i\omega \vec{B} = -i\omega \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times [\vec{E} + i\omega \vec{A}] = 0$$

$$\Longrightarrow \vec{E} + i\omega \vec{A} = -\nabla \phi$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{j} + i\omega\epsilon\mu \vec{E}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu \vec{j} + i\omega\epsilon\mu [-i\omega \vec{A} - \nabla \phi]$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{j} + \omega^2 \epsilon\mu \vec{A} - i\omega\epsilon\mu \vec{\nabla} \phi$$

$$\vec{\nabla} \underbrace{[\vec{\nabla} \cdot \vec{A} + i\omega\epsilon\mu\phi]}_{\text{make this zero:}} - \{\nabla^2 \vec{A} + \omega^2 \epsilon\mu \vec{A}\} = \mu \vec{j}$$

$$\phi = \frac{\vec{\nabla} \cdot \vec{A}}{-i\omega\epsilon\mu}$$

$$\nabla^2 \vec{A} + \omega^2 \epsilon\mu \vec{A} = -\mu \vec{j}$$

$\vec{i}$ component

$$\nabla^2 A_1 + \underbrace{\omega^2 \epsilon\mu A_1}_{} = -\mu j_1$$

if this were not here, Poisson equation.

$e^{i\omega t}$

$$i\omega = \frac{\partial}{\partial t}$$

$$\omega^2 = -\frac{\partial^2}{\partial t^2}$$

$$\nabla^2 A_1 - \epsilon\mu \frac{\partial^2 A_1}{\partial t^2} = -\mu j_1$$

$$\frac{\partial^2 A_1}{\partial t^2} = \frac{1}{\epsilon\mu} \nabla^2 A_1 + \frac{\mu}{\epsilon\mu} j_1$$

non-homogeneous wave equation.

Curves & line integrals.

Parametrize a curve:

straight line:

"time" or dummy parameter

$$\vec{R} = \vec{R}_0 + t\,\vec{V} \qquad -\infty < t < \infty$$

pt on line

$\parallel$ to line

circle

$$\vec{R} = a\cos\theta\,\vec{i} + a\sin\theta\,\vec{j} + \vec{R}_0 \qquad 0 \le \theta \le 2\pi$$

radius

parameter

center

Parametrize a surface

$$\vec{R} = \vec{R}(s, t) \quad \text{(it's 2-dimensional.)}$$

dummy variables

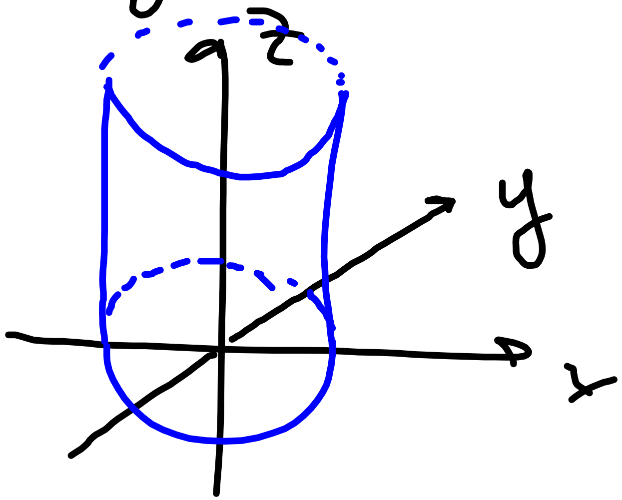
Plane:

$$\vec{R} = \vec{R}_0 + s\vec{A} + t\vec{B} \qquad -\infty < s < \infty \qquad -\infty < t < \infty$$

pt in plane

vectors in plane

Cylinder



$x = a\cos\theta$

$y = a\sin\theta$

$z = z$ free

$0 \le \theta \le 2\pi$

$0 \le z \le b$

radius $= a$

height $= b$

$$\vec{R} = a\cos t\,\vec{i} + b\sin t\,\vec{j} + s\vec{k}$$

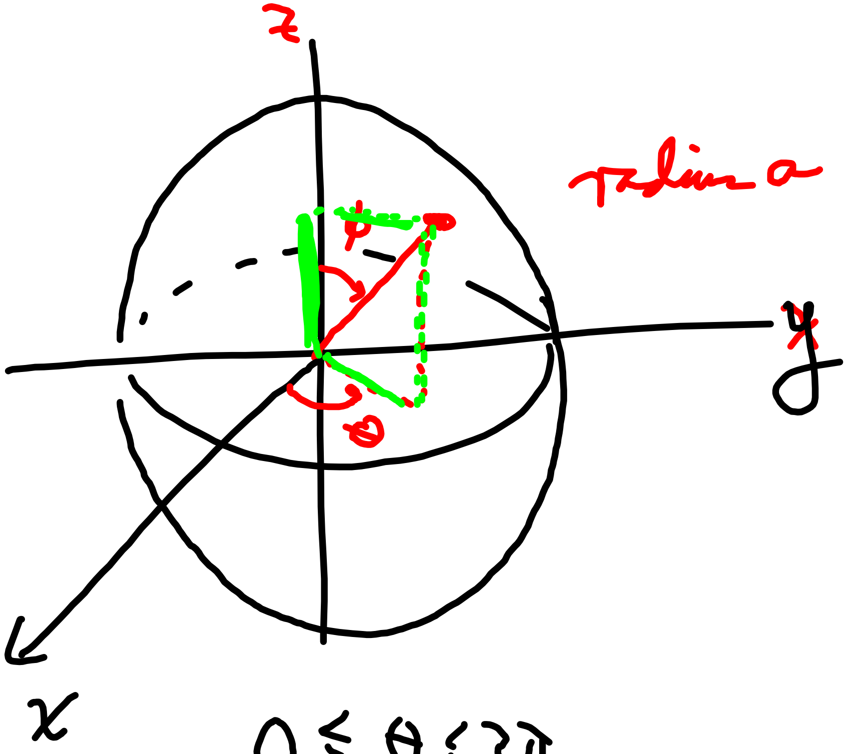
$0 \le t \le 2\pi$ $\quad$ $0 \le s \le b$

sphere

$z = a \cos \phi$

$x = a \sin \phi \cos \theta$

$y = a \sin \phi \sin \theta$



$0 \le \theta < 2\pi$

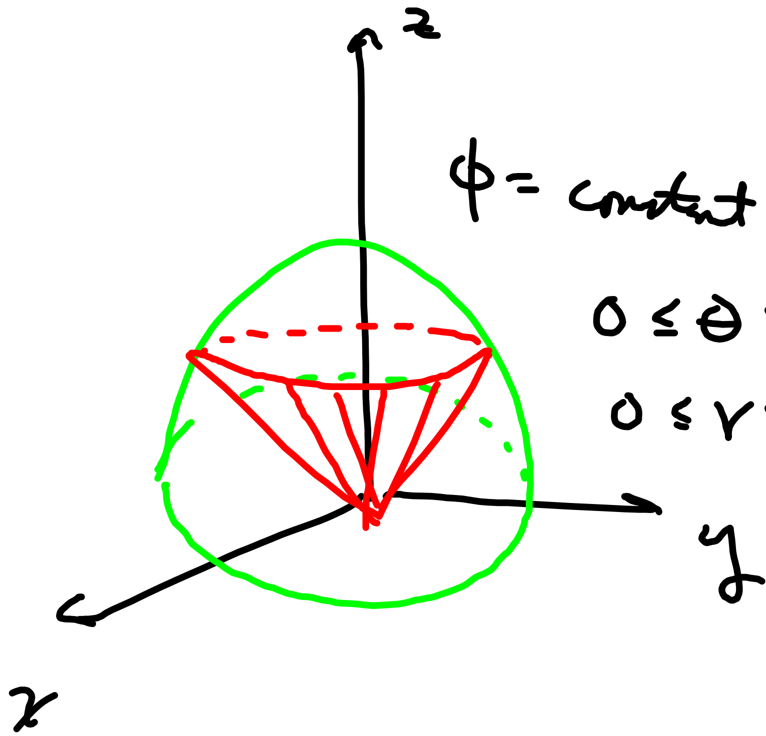
$0 \le \phi \le \pi$

$$\vec{R} = \vec{R}(s,t) = a \sin s \cos t \, \vec{i} + a \sin s \sin t \, \vec{j} + a \cos s \, \vec{k}$$

$s \leftrightarrow \phi$

$t \leftrightarrow \theta$

Cone



$\phi = \text{constant}$

$0 \le \theta \le 2\pi$

$0 \le r \le a$

$$\vec{R} = \vec{R}(s,t) = \cancel{a}^{\,r} \sin \cancel{s}^{\,\phi_0} \cos t \, \vec{i} + \cancel{a}^{\,r} \sin \cancel{s}^{\,\phi_0} \sin t \, \vec{j} + \cancel{a}^{\,r} \cos \cancel{s}_{\,\phi_0} \, \vec{k}$$

$0 \le t \le 2\pi$

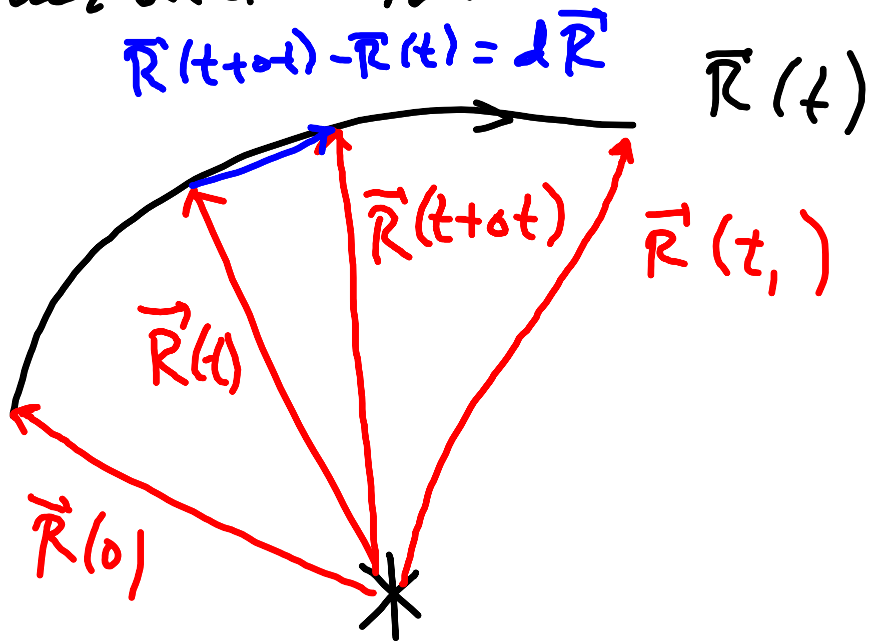
$0 \le r \le a$

$\cancel{s \leftrightarrow \phi}$

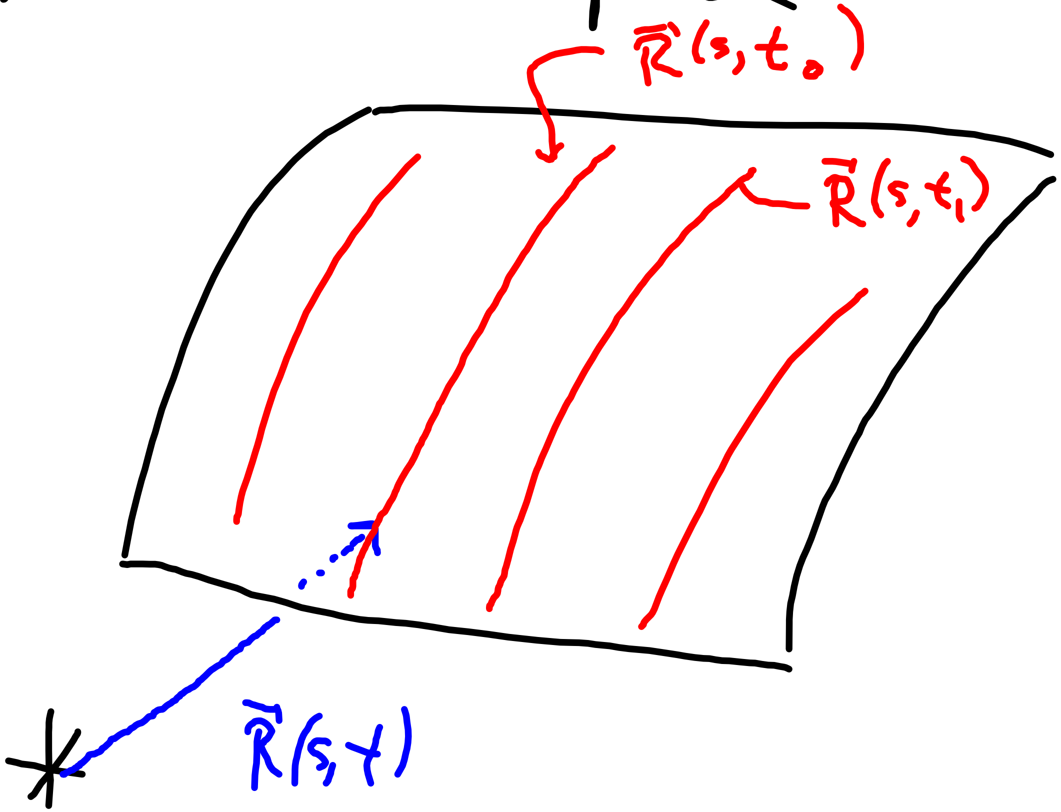
$\cancel{t \leftrightarrow \theta}$

$$\vec{R} = \vec{R}(r,t)$$

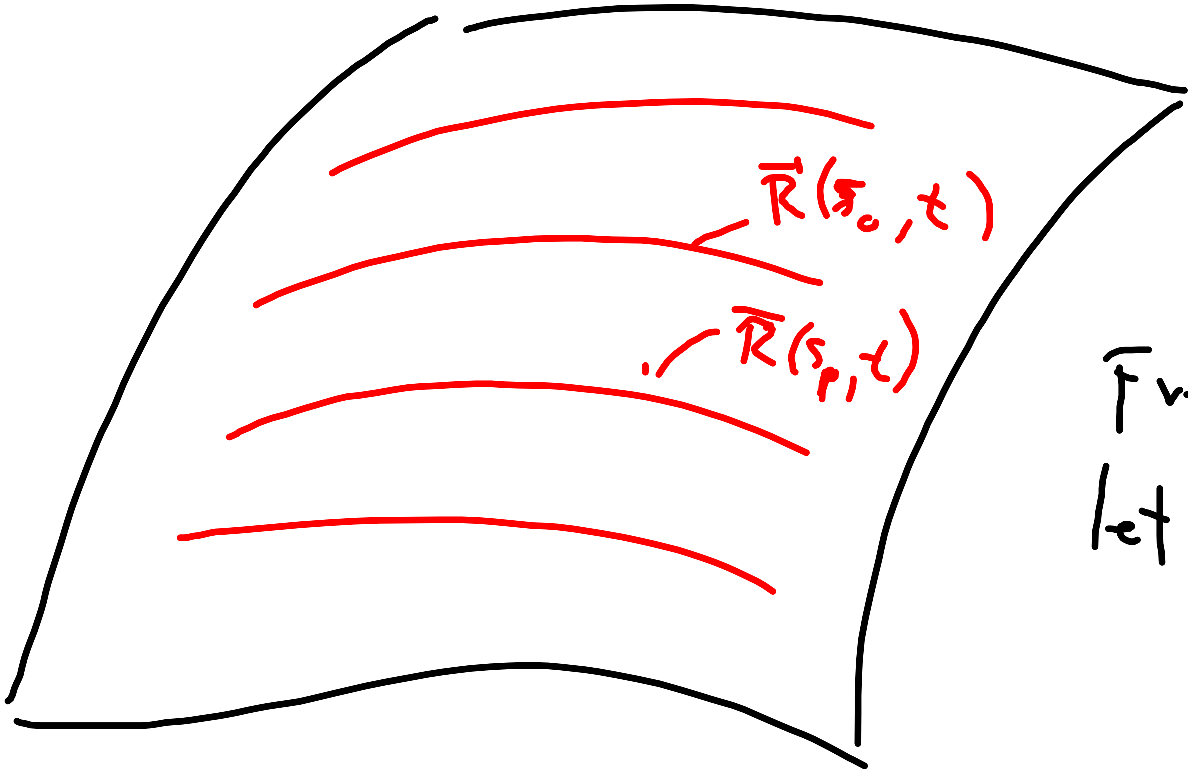
Calculus on a curve



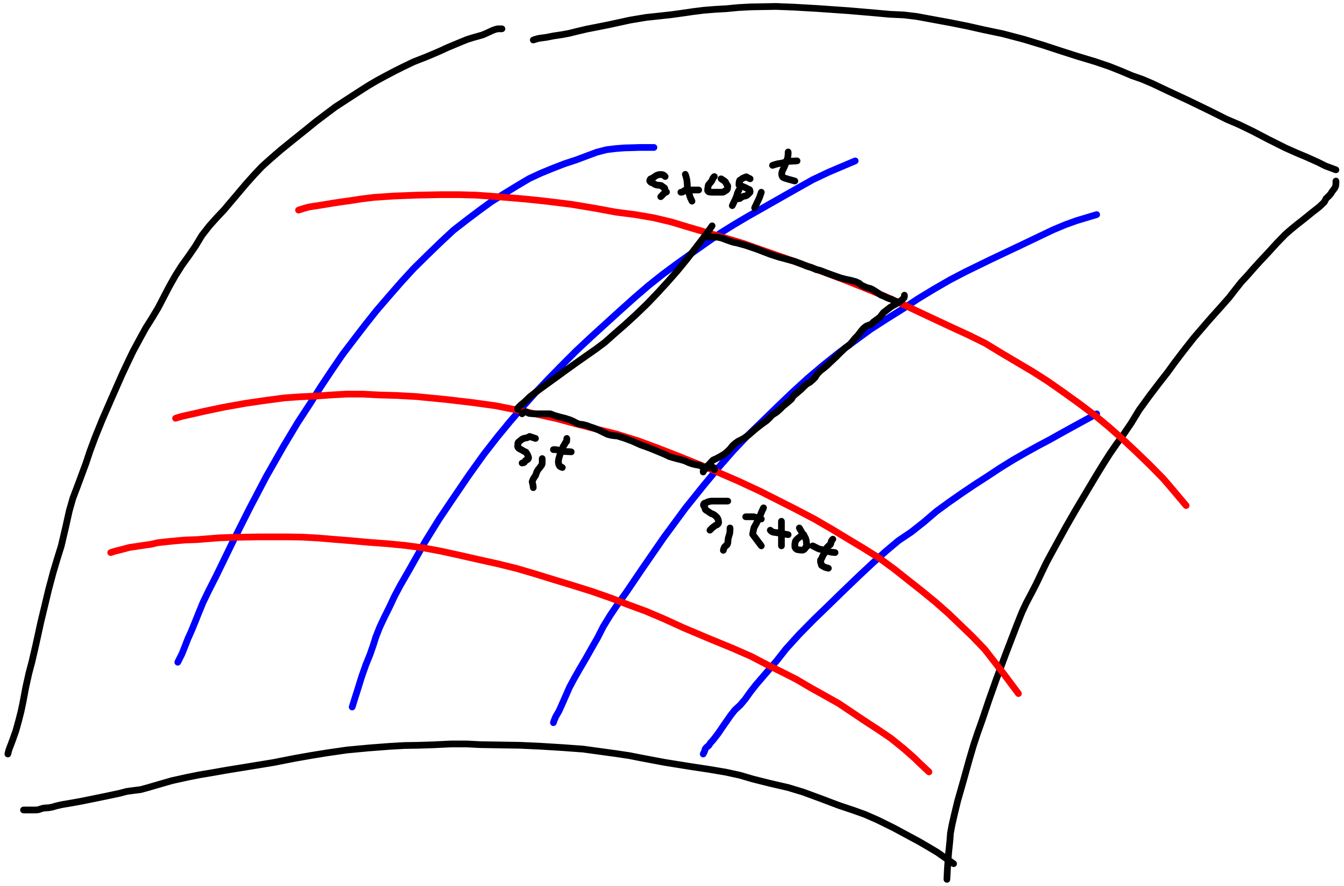
Calculus on a surface



Freeze t and let s vary.



Freeze s,
let t vary



s+Δs, t
s, t
s, t+Δt