Remember $\vec{\nabla} \times \vec{F} = 0 \iff \vec{F} = (\pm)\vec{\nabla}\phi$

How do you get ϕ, given $\vec{F}$?

Answer in practice: who cares?

The problem is to get $\vec{F}$, usually.

The point is, if you know $\vec{F}$ is a gradient, replace $\vec{F}$ throughout by $\nabla\phi$; now you only have find ONE unknown ϕ, instead of 3 (F_1, F_2, F_3).

Once you have ϕ, take $\vec{F} = \nabla\phi$.

Electrostatics

Much easier

Maxwell's eqs.

Delete the 4 pi here

$$\vec{\nabla}\cdot\vec{E} = 4\pi \cancel{\rho}\,\epsilon$$

So $\nabla^2\phi = -\frac{4\pi\rho}{\epsilon_0}$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = 0$$

So $\vec{E}$ is a gradient

$\vec{E} = -\nabla\phi$

$$\vec{\nabla}\times\vec{B} = 4\pi\epsilon\vec{j} + \frac{1}{\mu\epsilon}\frac{\partial\vec{E}}{\partial t} = 4\pi\epsilon\vec{j}$$

Right hand side should be
mu **j** + mu epsilon d**E**/dt

$$\vec{F} = \vec{\nabla} \times \vec{A} \implies \vec{\nabla} \cdot \vec{F} = 0$$

Today's <u>news</u> $\Longleftarrow$

$\vec{A}$ is called "vector potential."

In frequency domain: time dependent $e^{i\omega t}$,

maxwell's eq:

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho / \epsilon$$

?

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -i\omega \vec{B}$$

$$\vec{\nabla} \times \vec{B} = 4\pi \epsilon \vec{j} + \frac{1}{\mu\epsilon} \frac{\partial \vec{E}}{\partial t} = 4\pi\epsilon \vec{j} + \frac{i\omega}{\mu\epsilon} \vec{E}$$

?

$\vec{\nabla}\cdot\vec{B}=0$ so $\vec{B}=\vec{\nabla}\times\vec{A}$

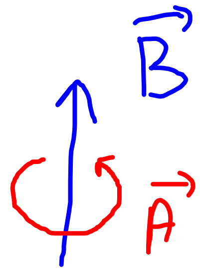
$$\vec{\nabla}\times\vec{E}=-i\omega\vec{B}=-i\omega\vec{\nabla}\times\vec{A}$$

so $\vec{\nabla}\times\left(\vec{E}+i\omega\vec{A}\right)=0$

$$\vec{E}+i\omega\vec{A}=-\nabla\phi$$

How do you ~~do~~ depict the vector potential?

Given $\vec{B}$, $\vec{B} = \vec{\nabla} \times \vec{A}$, sketch $\vec{A}$



So the lines-of-force of vector potential wrap around the lines-of-force of the field.

Why the vector potential is ambiguous:

given $\vec{B} = \vec{\nabla} \times \vec{A}$

Suppose you add $\vec{\nabla}\phi$ to $\vec{A}$

$$\vec{B} = \vec{\nabla} \times (\vec{A} + \nabla\phi)$$

SNIDER's thm. You can add a gradient to zero out any particular component of $\vec{A}$.

Lorentz's thm. You can add a gradient which makes it easy to solve Maxwell's equations, Two-dimensional fields $\vec{F} = F_1 \vec{i} + F_2 \vec{j}$

more than that, $\vec{F} = F_1(x,y)\vec{i} + F_2(x,y)\vec{j}$

Conservative - 2D fields:

$$\vec{\nabla} \times \vec{F} = \vec{0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & 0 \end{vmatrix}$$

$$= \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \vec{k} \ldots$$

Conclusion: $\vec{F} = \vec{\nabla}\phi$

$$F_1 = \frac{\partial \phi}{\partial x} \qquad F_2 = \frac{\partial \phi}{\partial y}$$

Zero divergence case.

$$\cancel{\vec{F} = F_1(x,y)\vec{i} + F_2(x,y)\vec{j}}$$

$$\vec{F} = f_1(x,y)\vec{i} + f_2(x,y)\vec{j}$$

$$\vec{\nabla}\cdot\vec{F} = 0 = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} = 0$$

conclusion $\vec{F} = \vec{\nabla}\times\vec{G}$

Here's a trick for finding $\vec{G}$ in this case

Observe $\vec{\nabla}\times\left[f_2\vec{i} - f_1\vec{j}\right]$ is zero:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ f_2 & -f_1 & 0 \end{vmatrix} = \left(-\frac{\partial f_1}{\partial x} - \frac{\partial f_2}{\partial y}\right)\vec{k}$$

note: $(f_2\vec{i} - f_1\vec{j})$ is $(f_1\vec{i} + f_2\vec{j})\times\vec{k}$

So $\left[f_2\vec{i} - f_1\vec{j}\right] = \nabla\psi \qquad f_2 = \frac{\partial\psi}{\partial x},\ f_1 = -\frac{\partial\psi}{\partial y}$

now take $\vec{\nabla}\times(-\psi\vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & -\psi \end{vmatrix} = \vec{i}f_1 + \vec{j}f_2$

" original, divergenceless field.

Bottom line:

If $\vec{F}$ is 2-D & $\vec{\nabla} \times \vec{F} = 0$

then $\vec{F} = \nabla \times \vec{G}$

where $\vec{G} = \psi \vec{K}$ and ψ is

the scalar potential for $\vec{K} \times \vec{F}$.