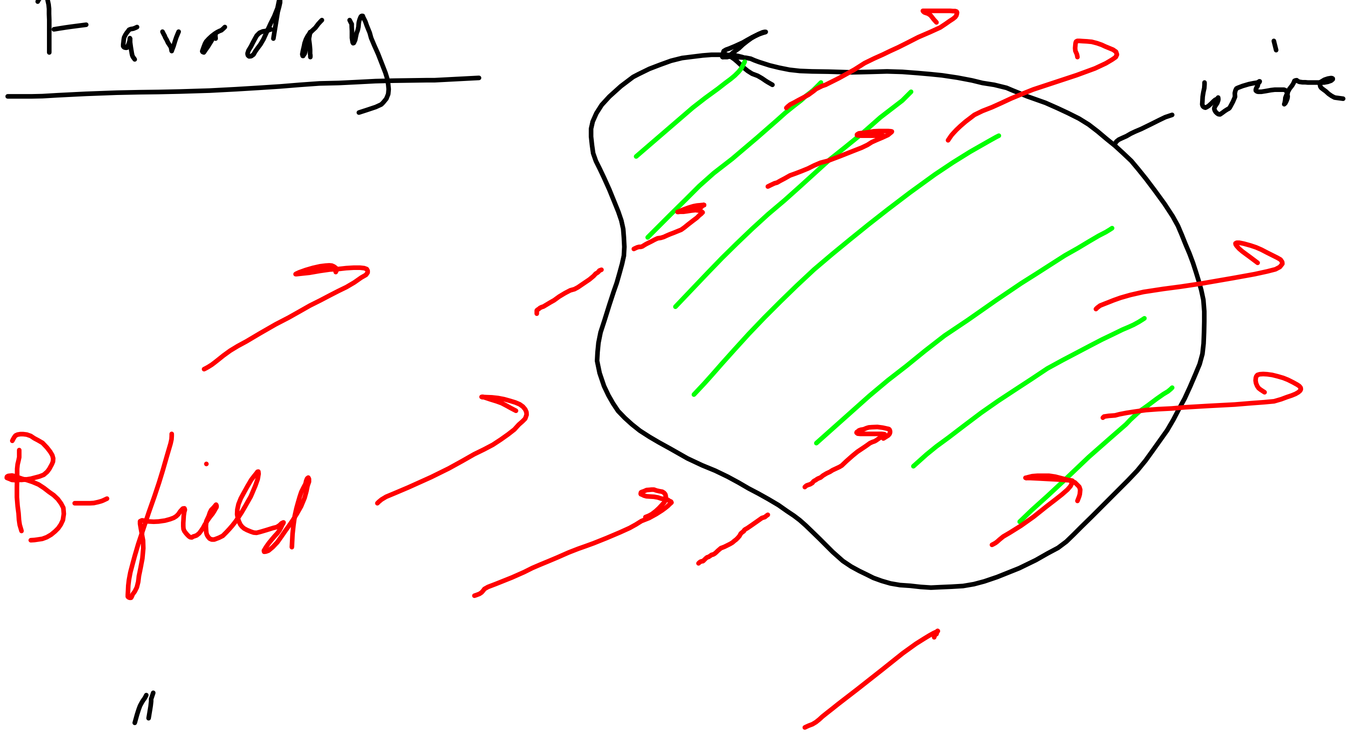
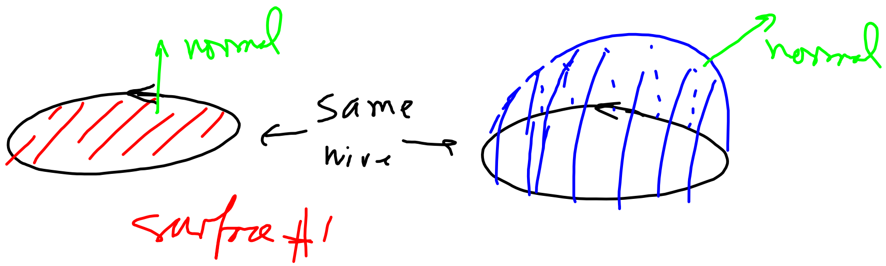
# Faraday



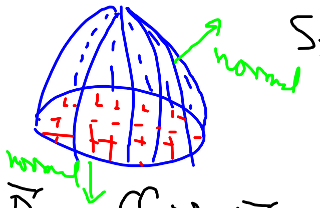

"Magnetic Flux" $= \iint_S \vec{B} \cdot d\vec{S}$

$$\frac{d}{dt} \underbrace{\iint}_{\text{surface}} \vec{B} \cdot d\vec{S} = \underbrace{\oint}_{\text{wire}} \vec{E} \cdot d\vec{r}$$




It better be true that $\iint_{S\#1} \vec{B}\cdot d\vec{S} = \iint_{S\#2} \vec{B}\cdot d\vec{S}$

Solid




outflow of $\vec{B} = \iint_{\text{whole surface}} \vec{B}\cdot d\vec{S}_{\text{outer}} = \iint \vec{B}\cdot d\vec{S}_{\text{out}} + \iint \vec{B}\cdot d\vec{S}_{\text{out}}$

Does $\iint_{\text{solid.}} \vec{B}\cdot d\vec{S} = 0$?

This is Faraday flux #1

This is (−) Fa. Flux #2

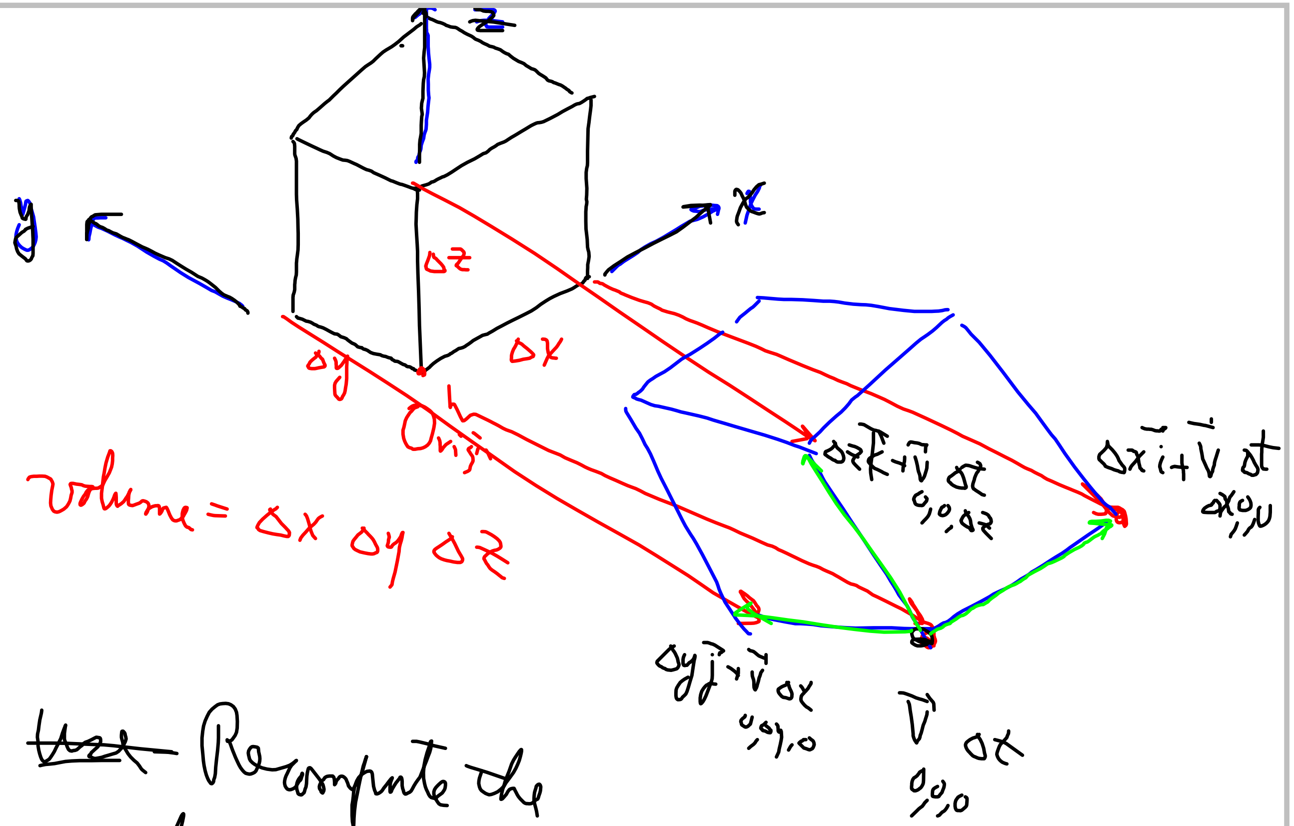
Div. Thm. $\iint_{\text{solid}} \vec{B}\cdot d\vec{S} = \iiint \underbrace{\vec{\nabla}\cdot\vec{B}}_{=0} \, d\text{volume}$ (Maxwell)

# Fluid-flow

## Velocity field

velocity $\vec{V} = (\vec{R})$

divergence of velocity $\vec{\nabla} \cdot \vec{V} = \frac{d\,\text{volume}}{dt} \frac{1}{\text{volume}}$




~~Use~~ Recompute the volume using the triple scalar product of the 3 sides, you will find

$$\frac{\Delta V}{V} = (\vec{\nabla}\cdot\vec{V})\,\Delta t \qquad \frac{dV}{dt}\frac{1}{V} = \nabla\cdot\vec{V}$$

# Transport Theorems.

Flux transport theorem.

$$\iint \vec{B} \cdot d\vec{S}$$

$$\frac{\partial \vec{B}}{\partial t} \neq 0 \qquad \text{Surface is moving}$$

# Scalar potential.

$$\text{vector } \vec{F}(\vec{R}) = \nabla \phi(\vec{R})$$

First check that $\vec{\nabla} \times \vec{F} = \vec{0}$

$$x\vec{i} + z\vec{j} + y\vec{k} = \nabla\left(\frac{x^2}{2} + yz\right)$$

Try

$$\phi = \int x\,dx + \int z\,dy + \int y\,dz \quad \text{wrong}$$

$$x^2/2 + zy + yz \quad = x^2/2 + 2yz$$

$\cancel{x\,\vec{i} + y}$

$$x\,\vec{i} + z\,\vec{j} + (y+z^2)\,\vec{k} = \vec{\nabla}\phi$$

need $\dfrac{\partial\phi}{\partial x} = x \longleftarrow \phi = \dfrac{x^2}{2} + \ldots z \ldots$

$\dfrac{\partial\phi}{\partial y} = z \longleftarrow \phi = \dfrac{x^2}{2} + zy + \ldots z \ldots$

$\dfrac{\partial\phi}{\partial z} = y + z^2 \longleftarrow 0 + y + \cancel{\ldots}$

$$\phi = \frac{x^2}{2} + yz + z^3/3$$

$$\vec{F} = x\vec{i} + z\vec{j} + (2y + z^2)\vec{k}$$

$\nabla \times \vec{F} \neq \vec{0}$, you won't make it,

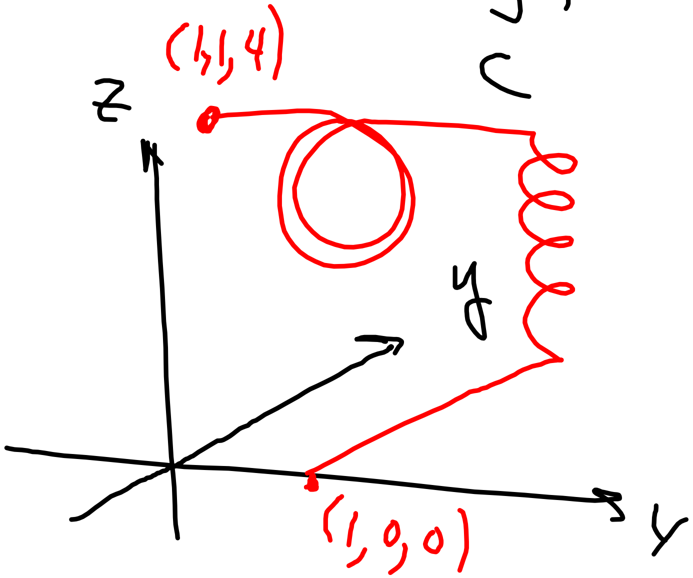
$$\phi = \frac{x^2}{2} + zy + \psi(z)$$

$$\frac{\partial \phi}{\partial z} = 2y + z^2 = 0 + y + (y + z^2)$$

## Example:

$$\vec{F} = x\,\vec{i} + z\,\vec{j} + (y+z^2)\,\vec{k}$$

Work out $\int_C \vec{F}\cdot d\vec{R}$




Sol'n:

$$\vec{F} = \vec{\nabla}\left(\frac{x^2}{2}\cancel{\vec{i}} + yz\cancel{\vec{j}} + \frac{z^3}{3}\cancel{\vec{k}}\right)$$

$$\int \vec{F}\cdot d\vec{R} = \phi(\text{end}) - \phi(\text{start})$$

$$= \left(\frac{x^2}{2} + yz + \frac{z^3}{3}\right)\Bigg|_{(1,0,0)}^{(1,1,4)}$$

Find the vector potential:

$$\vec{F}(x,y) = (x+y)\vec{i} + (-y+x)\vec{j}$$

2-D field only $x, y$ only $\vec{i}, \vec{j}$

There is a vector potential of the form $0\vec{i} + 0\vec{j} + \psi\vec{k}$.

$$(x+y)\vec{i} + (-y+x)\vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & \psi \end{vmatrix}$$

$$x+y = \frac{\partial \psi}{\partial y} \qquad \psi = xy + \frac{y^2}{2} + \ldots x \ldots \quad -x^2/2$$

$$-y+x = -\frac{\partial \psi}{\partial x} \qquad = -y + 0$$

answer: $0\vec{i} + 0\vec{j} + \left(xy + \frac{y^2}{2} - \frac{x^2}{2}\right)\vec{k}$