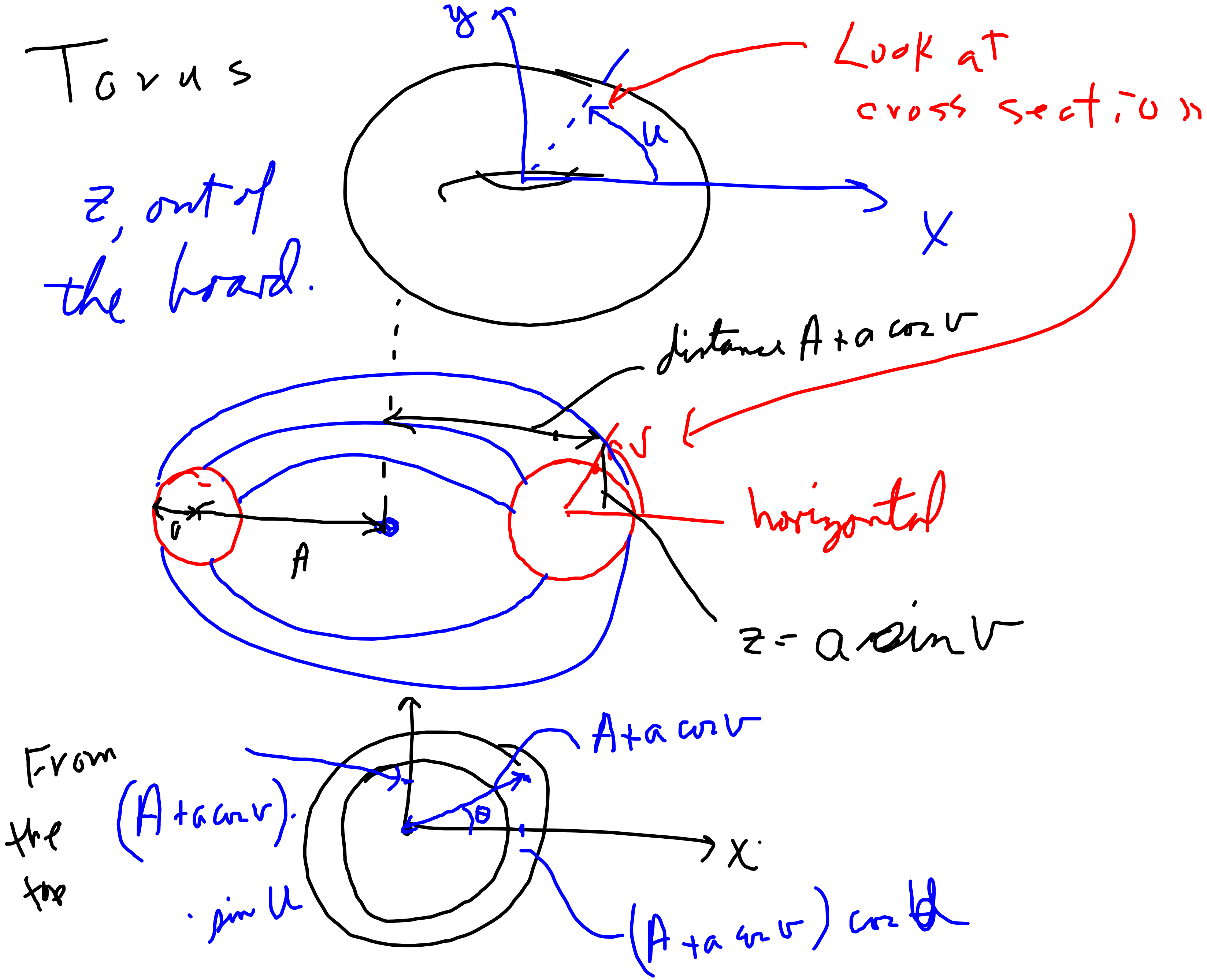
Torus

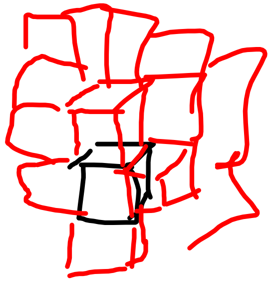


y
u
x
Look at cross section
z, out of the board.
distance A + a cos v
v
horizontal
a
A
z = a sin v
From the top
(A + a cos v) · sin u
A + a cos v
θ
x
(A + a cos v) cos θ

Divergence Theorem.

$$\boxed{\iiint_{\text{volume}} \vec{\nabla}\cdot\vec{F}\;dx\,dy\,dz = \iint_{\text{enclosing surface}} \vec{F}\cdot d\vec{S}}$$

Divergence = outflux per unit volume



$\sum_{\text{all cubes}}$ outflux

= net outflux through bndry faces.

Σ outfluxes = outflux out the exterior (interior faces cancel).

$$d\vec{S} = \frac{\partial \vec{R}}{\partial u}\times\frac{\partial \vec{R}}{\partial v}\,du\,dv$$

$$= \vec{n}_{\text{unit normal}}\; d(\text{area})$$

outward

Prob. 4 Final 03a.

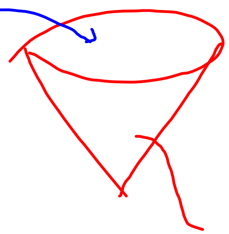
$x = \rho \cos\theta$

$y = \rho \sin\theta$

$z = 2$

$0 \le \theta \le 2\pi$

$0 \le \rho \le 2$



$z = r$

$x = r\cos\theta$

$y = r\sin\theta$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$\iint \vec{F}\cdot d\vec{S} = \iiint \vec{\nabla}\cdot\vec{F}\; d\text{volume}$$

$$\vec{F} = y^2\vec{i} + z\vec{K}$$

$$\vec{\nabla}\cdot\vec{F} = 1$$

$$= \iiint 1\; d\text{volume}$$

$$= \frac{1 \times \pi \times 2^2 \times 2}{3}$$

Lid: $\vec{R}_L = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + 2\vec{K}$ (blue)

Cone $\vec{R}_C = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} + r\vec{K}$ (red)

$$\text{Lid outflow} = \int_{\rho=0}^{2}\int_{\theta=0}^{2\pi} \vec{F}\cdot \frac{\partial \vec{R}_L}{\partial\rho} \times \frac{\partial \vec{R}_L}{\partial\theta}\, d\theta\, dr$$

$$+ \text{Cone outflow} = \int_{r=0}^{2}\int_{\theta=0}^{2\pi} \vec{F}\cdot \frac{\partial \vec{R}_c}{\partial r} \times \frac{\partial \vec{R}_c}{\partial\theta}\, d\theta\, dr$$

$8\pi/3$

Final 02b #7.

On top $\vec{n} = \vec{k}$, $\vec{F}\cdot\vec{n} = z = 2$,

$$\iint \vec{F}\cdot\vec{n}\, d\text{area} = 2\cdot\pi\cdot 1^2 = 2\pi$$

On bottom $\cancel{\vec{n}} \; \vec{n} = -\vec{k}$, $\vec{F}\cdot\vec{n} = -z = -1$

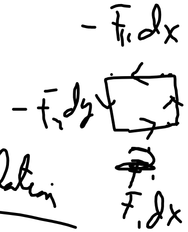
$$\iint \vec{F}\cdot\vec{n}\, dS = \iint \vec{F}\cdot\vec{n}\, d\,\cancel{\text{area}}\ \text{area} = (-1)\pi 1^2 = -\pi$$

$$\text{Top} + \text{Bottom} + \text{Side Flux} = \iiint \underbrace{\nabla\cdot\vec{F}}_{0}\, d\,\text{volume}$$

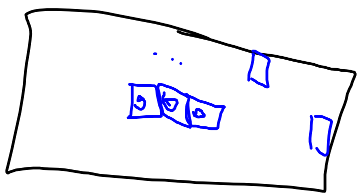
$\therefore$ Side flux $= -\pi$

Stokes' Theorem.

$\vec{\nabla} \times \vec{F} = \dfrac{\text{circulation}}{\text{area}}$



$= \int \vec{F} \cdot d\vec{R}$ = circulation around



$\iint (\vec{\nabla} \times \vec{F}) \cdot \vec{n} \, d\,\text{area} =$

$\oint \vec{F} \cdot d\vec{r}$

boundary of curve

$$\iint_{\text{surface}} (\vec{\nabla} \times \vec{F}) \cdot d\vec{S} = \oint_{\text{rim of surface}} \vec{F} \cdot d\vec{r}$$

$$d\vec{S} = \frac{\partial \vec{R}}{\partial u} \times \frac{\partial \vec{R}}{\partial v} \, du \, dv$$

$$= \vec{n} \, d\,\text{area}$$

$$\iiint \vec{\nabla} \cdot \vec{F} \, d\,\text{volume} = \iint \vec{F} \cdot d\vec{S}$$

$$\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \int \vec{F} \cdot d\vec{r}$$

$$\int \vec{\nabla}\phi \cdot dr = \phi(\text{end}) - \phi(\text{start})$$

Don't think about this.

Div. Theorem $\iiint \vec{\nabla} \cdot (\vec{\nabla} \times \vec{G}) \, d\,\text{volume} = \iint (\nabla \times \vec{G}) \cdot d\vec{S}$

$$\underset{0}{\parallel}$$