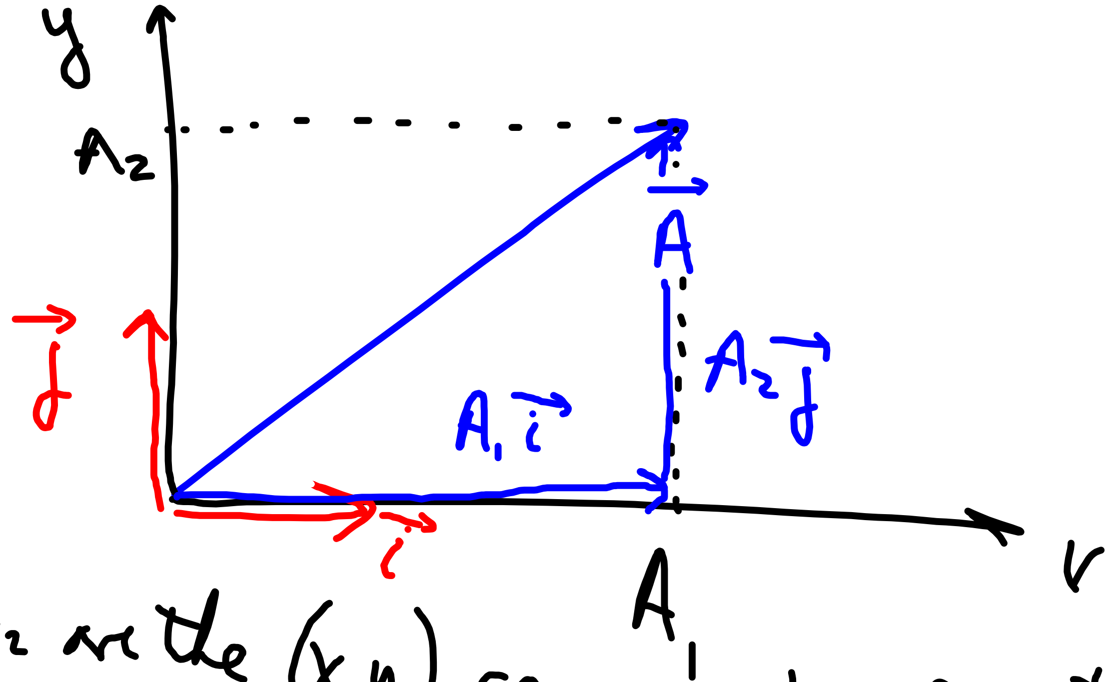
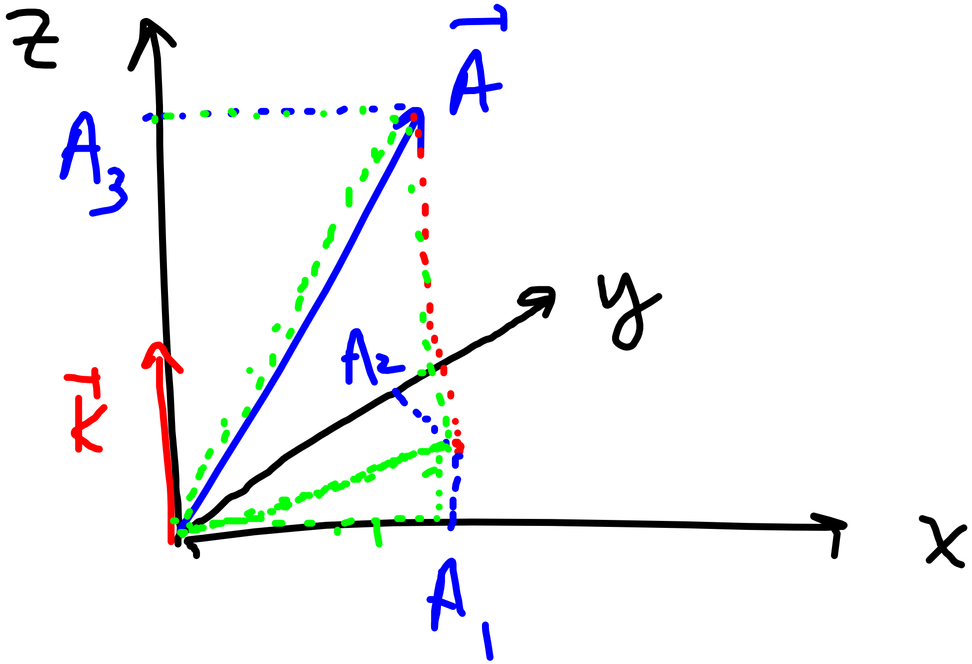
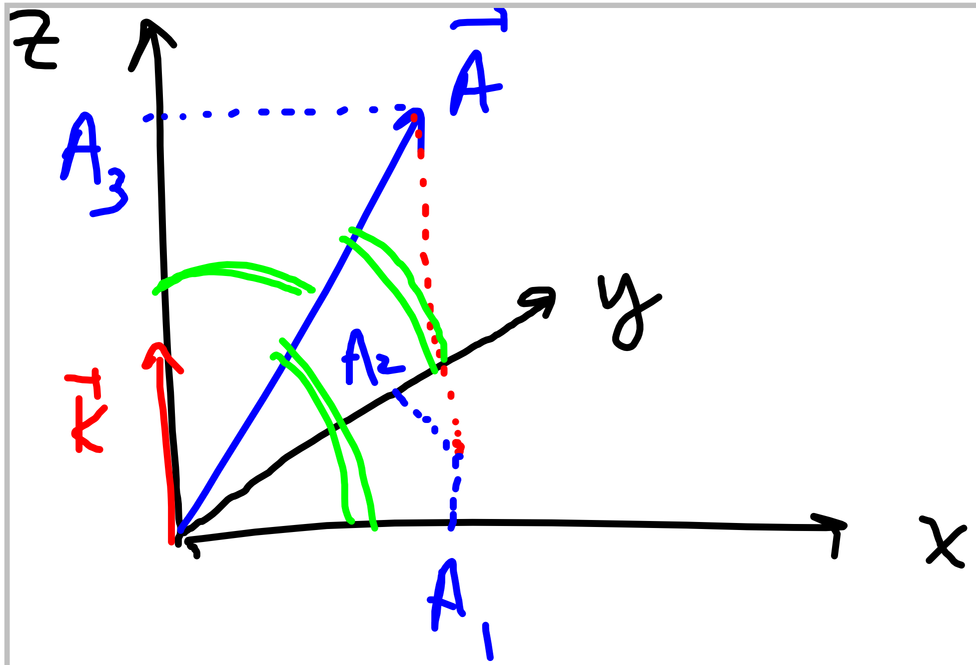


A_1, A_2 are the (x, y) components of vector.

$$\vec{A} = A_1\vec{i} + A_2\vec{j}$$



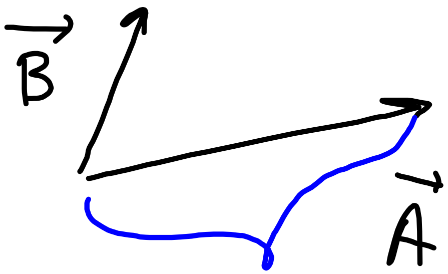
$$\vec{A} = A_1\vec{i} + A_2\vec{j} + A_3\vec{k}$$



direction angles

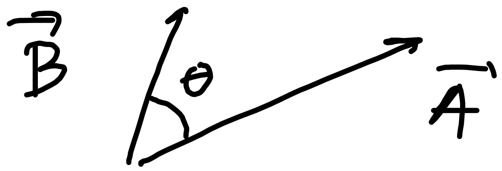
(Temporarily skip Sec. 1.8.)

"Dot product" of vectors.



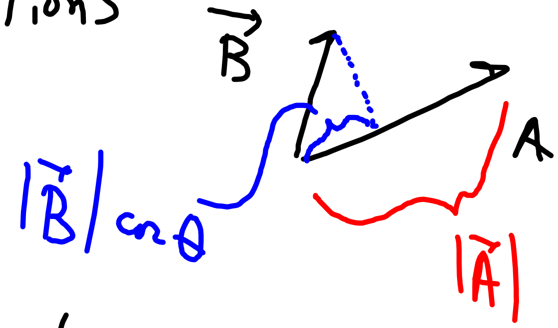
$|\vec{A}|$ = length of $\vec{A}$

intensity of A

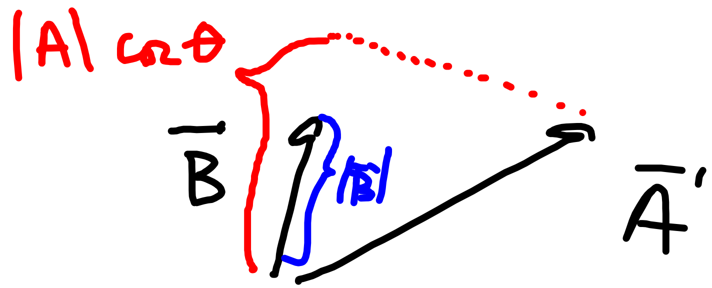


$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

Interpretations

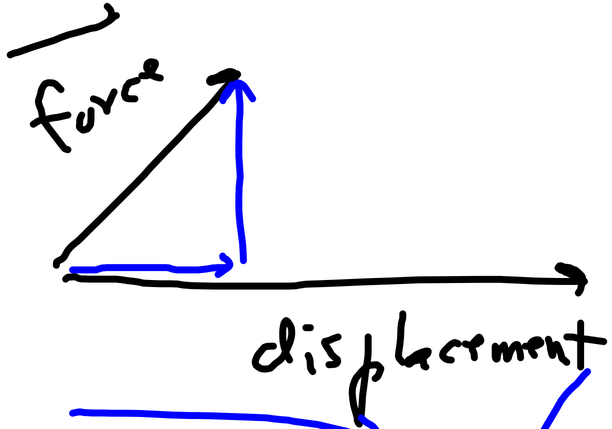


$$\vec{A} \cdot \vec{B} = (\text{length of } \vec{A})(\text{part of } \vec{B} \text{ that's} \parallel \text{to } \vec{A})$$



$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = (\text{length of } \vec{B})(\text{part of } \vec{A} \parallel \text{to } \vec{B})$$

Mechanical interpretation:



$$\overrightarrow{\text{force}} \cdot \overrightarrow{\text{displacement}} = (\text{effective component of force}) \cdot (\text{displacement})$$

$$= \text{"work"}$$

Fundamental Formula.

$$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta$$

$$\vec{A} = A_1 \vec{i} + A_2 \vec{j} + A_3 \vec{k}$$

$$\vec{B} = B_1 \vec{i} + B_2 \vec{j} + B_3 \vec{k}$$

$$\boxed{\vec{A} \cdot \vec{B} = A_1 B_1 + A_2 B_2 + A_3 B_3} = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

Proved in §1.6.

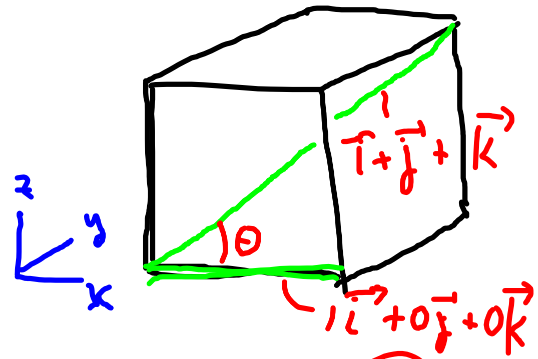
"Dot" = "Scalar" = "Inner" product

Application. $|\vec{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$

$$= \sqrt{\vec{A} \cdot \vec{A}}$$

Application.

unit cube

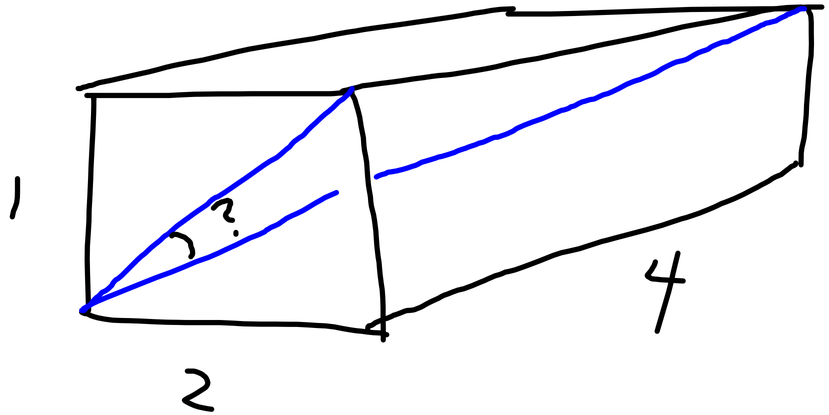


What is $\angle$ angle between the edge + the body diagonal?

$$\vec{A} \cdot \vec{B} = |A|\,|B| \cos\theta = A_1 B_1 + A_2 B_2 + A_3 B_3$$

$$|1| \cdot |\sqrt{3}| \cos\theta = 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1$$

$$\cos\theta = \frac{1}{\sqrt{3}} \quad \theta = \cos^{-1} \frac{1}{\sqrt{3}}$$



$$\vec{A} = (\vec{E} + \vec{F})$$

$$\vec{B} = (2\vec{G} - 3\vec{H})$$

$$\vec{A} \cdot \vec{B} = (\vec{E} + \vec{F}) \cdot (2\vec{G} - 3\vec{H})$$

$$= 2\vec{E} \cdot \vec{G} + 2\vec{F} \cdot \vec{G} - 3\vec{E} \cdot \vec{H} - 3\vec{F} \cdot \vec{H}$$

$$\vec{A} = A_1\vec{i} + A_2\vec{j} + A_3\vec{k}$$

$$\vec{B} = B_1\vec{i} + B_2\vec{j} + B_3\vec{k}$$

$$\vec{A}\cdot\vec{B} = A_1B_1\overset{=1}{(\vec{i}\cdot\vec{i})} + A_1B_2\overset{0}{(\vec{i}\cdot\vec{j})} + A_2B_1\,\vec{j}\cdot\vec{i}$$

$$+ A_2B_2\underset{=1}{(\vec{j}\cdot\vec{j})} +$$

$$= A_1B_1 + A_2B_2 + A_3B_3$$

$$\vec{A} = 4\vec{i} + 7\vec{j}$$

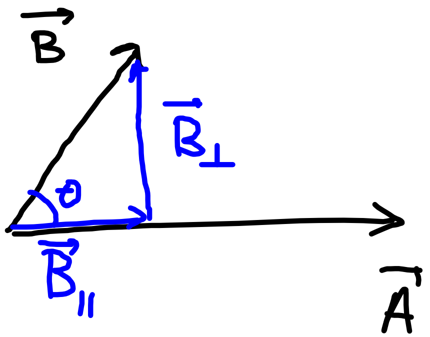
$$\vec{B} \perp A; \quad \vec{B} = \underline{7}\,\vec{i} + \underline{(-4)}\,\vec{j}$$

$$\vec{A} = a\vec{i} + b\vec{j}$$

$$\perp \text{to } \vec{A} = b\vec{i} - a\vec{j}$$

$\vec{A} = 3\vec{i} + 4\vec{j} + 7\vec{k}$

$\perp \vec{A}: \quad 4\vec{i} - 3j + 0\vec{k}$



Find $\vec{B}_\parallel$ first; then $\vec{B}_\perp = \vec{B} - \vec{B}_\parallel$

Do it in pieces.

$$\vec{B}_\parallel = \left(\underset{\text{length}}{\overset{|\vec{B}|\cos\theta}{\text{correct}}}\right)\left(\begin{array}{c}\text{correct}\\ \text{direction}\end{array}\right)$$

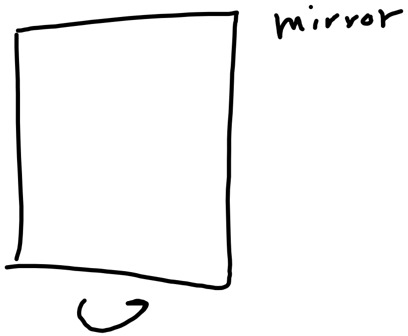
$$= \frac{\vec{B}\cdot\vec{A}}{|\vec{A}|}\ \frac{\vec{A}}{|\vec{A}|}$$

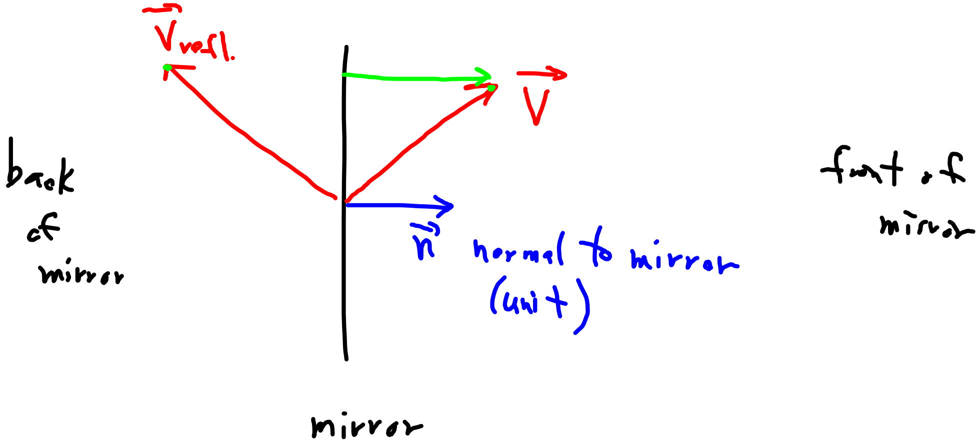
(to make this a unit vector)

$$\vec{B}_\parallel = \frac{\vec{B}\cdot\vec{A}}{\vec{A}\cdot\vec{A}}\vec{A}$$

$$\vec{B}_\perp = \vec{B} - \vec{B}_\parallel$$

Application: mirror reflection.





$$\vec{V}_{refl} = \vec{V} - 2 \times \text{part of } \vec{V} \text{ that's } \parallel \text{ to } \vec{n}$$

$$= \vec{V} - 2\frac{\vec{V}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n}$$

(1)